## $\S9.3\ (Part\ 1)$ : Branch & Bound Method for Pure IPs

1.] Solve the following IP using the Branch-and-Bound method:

Maximize: 
$$z = 5x_1 + 4x_2$$

Subject to: 
$$\begin{aligned} x_1 + x_2 &\leq 5\\ 10x_1 + 6x_2 &\leq 45\\ x_1, x_2 &\geq 0, x_1, x_2 \text{ integer} \end{aligned}$$

a.) Solve the LP Relaxation problem:



 $b.)\,$  Label on the graph above the feasible space for the IP. What seems to be the optimal solution for the IP?

2.] Using  $x_1$  as the branching variable, divide the solution space into two regions: one with  $x_1 \leq 3$  and  $x_1 \geq 4$ . Write down the corresponding LPs for each subregion, labeling the LP with  $x_1 \leq 3$  as LP2 and the LP with  $x_1 \geq 4$  as LP3. Solve LP2 first, establish a lower bound on z, then solve LP3.



- 3.] What is the solution to the IP?
- 4.] Discuss the procedure if we would have solved LP3 first.