§9.3 (PART 1): BRANCH & BOUND METHOD FOR PURE IPS

1.] Solve the following IP using the Branch-and-Bound method:

$$\text{Maximize:} \quad z = 5x_1 + 4x_2$$

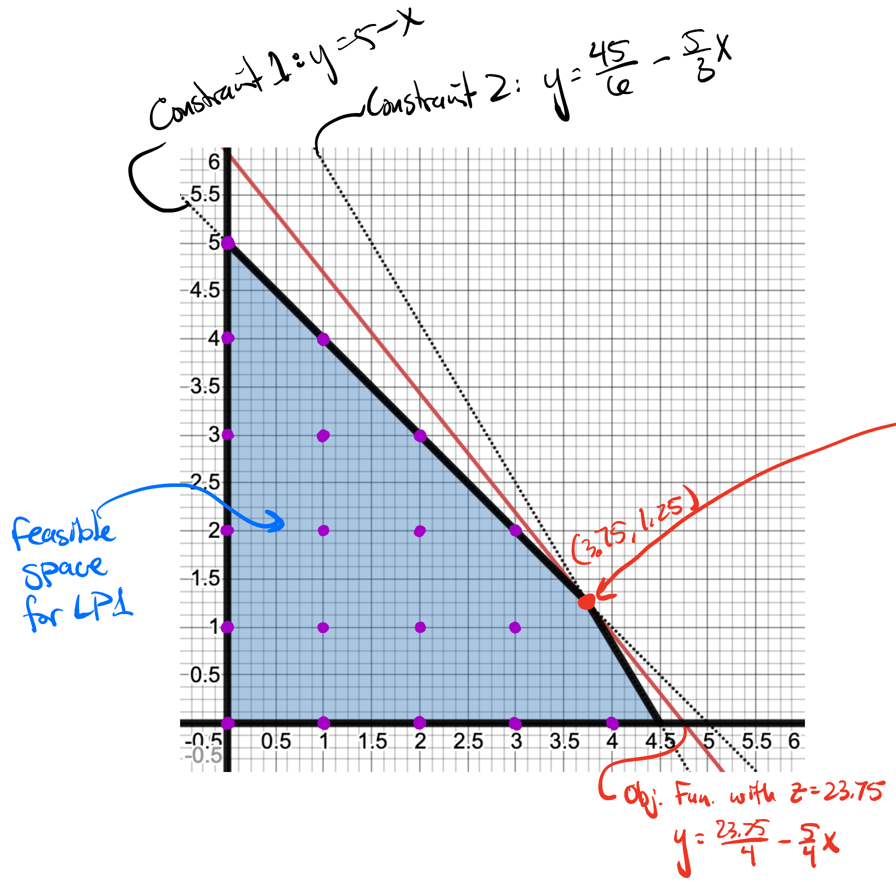
$$\begin{aligned} \text{Subject to:} \quad x_1 + x_2 &\le 5 \\ 10x_1 + 6x_2 &\le 45 \\ x_1, x_2 \ge 0, x_1, x_2 &\text{ integer} \end{aligned}$$

a.) Solve the LP Relaxation problem:

LP1: Relax integer constraints

Maximize $Z = 5X_1 + 4X_2$

Subject to
$X_1 + X_2 \le 5$
$10X_1 + 6X_2 \le 45$
$X_1, X_2 \ge 0$



Optimal Solution: $X_1 = 3.75$, $X_2 = 1.25$

Max $Z = 23.75$

Initial Iteration of Branch-and-Bound:

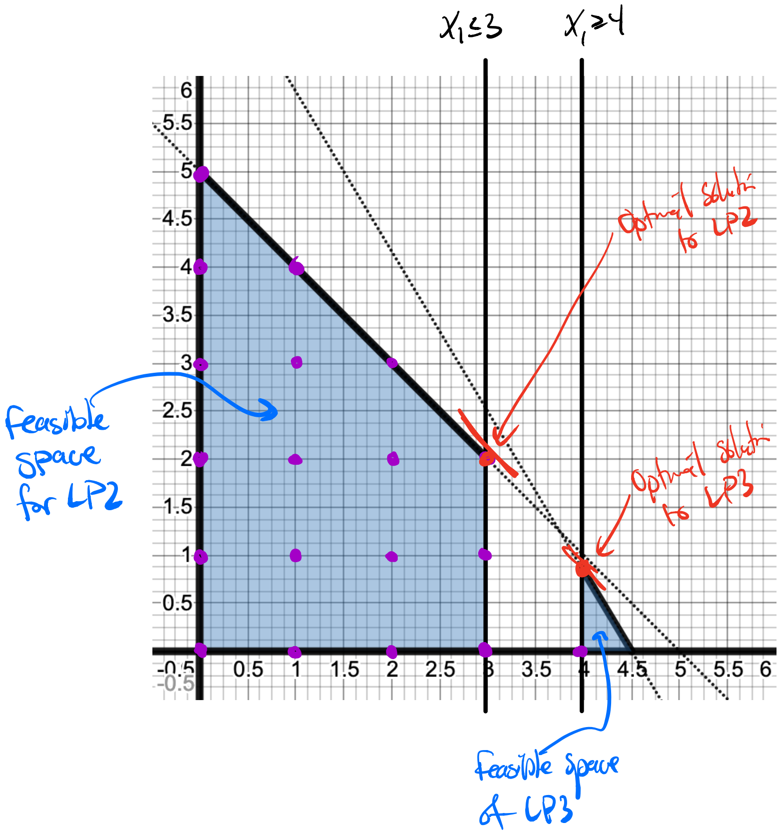
LP1
$X_1 = 3.75$, $X_2 = 1.25$
$Z = 23.75$

b.) Label on the graph above the feasible space for the IP. What seems to be the optimal solution for the IP?

⁚⁚ - Feasible Space of original IP

* From the graph alone, the likely solution is $X_1 = 3$, $X_2 = 2$ since this point is the closest feasible point to the obj. function that is integer-valued.

2.] Using x_1 as the branching variable, divide the solution space into two regions: one with $x_1 \leq 3$ and $x_1 \geq 4$. Write down the corresponding LPs for each subregion, labeling the LP with $x_1 \leq 3$ as LP2 and the LP with $x_1 \geq 4$ as LP3. Solve LP2 first, establish a lower bound on z, then solve LP3.



LP2: (Solve this first)

Maximize $Z = 5X_1 + 4X_2$

Subject to

$$X_1 + X_2 \leq 5$$
$$10X_1 + 6X_2 \leq 45$$
$$X_1 \leq 3$$
$$X_1, X_2 \geq 0$$

LP3:

Maximize $Z = 5X_1 + 4X_2$

Subject to

$$X_1 + X_2 \leq 5$$
$$10X_1 + 6X_2 \leq 45$$
$$X_1 \geq 4$$
$$X_1, X_2 \geq 0$$

LP2
$X_1 = 3$, $X_2 = 2$
$Z = 23$

- Solution to LP2 is integer-valued with $Z = 23$.
- $Z = 23$ is a Lower Bound on our original IP.
- LP2 is FATHOMED
  (it cannot yield a better IP solution)
- No further branching required.

Now examine LP3:

Note: Since LP1 has solution $Z = 23.75$, and $Z = 5X_1 + 4X_2$, there's no way LP3 can produce an integer solution better than $Z = 23$.

- LP3 is FATHOMED.

3.] What is the solution to the IP?

IP Solution: $X_1 = 3, X_2 = 2, Z = 23$.

4.] Discuss the procedure if we would have solved LP3 first.

- If LP3 is solved first, we would have to continue branching until all sub problems are FATHOMED.

- Solve LP2 first: LP1 → LP2 → LP3 → DONE
- Solve LP3 first:

LP1 → LP3 → LP5 → LP4 → LP7 → LP6 → LP2 → DONE