§9.2 (PART 1): FORMULATING INTEGER PROGRAMMING PROBLEMS

1.] SET COVERING: There are six cities in Kilroy County. The county must determine where to build fire stations. The county wants to build the minimum number of fire stations needed to ensure that at least one fire station is with 15 minutes (driving time) of each city. The times (in minutes) required to drive between the cities in Kilroy County are given in the table below. Formulate an IP that will tell Kilroy how many fire stations should be built and where they should be located.

Time Required to Travel between Cities in Kilroy County						
	City 1	City 2	City 3	City 4	City 5	City 6
City 1	0	10	20	30	30	20
City 2	10	0	25	35	20	10
City 3	20	25	0	15	30	20
City 4	30	35	15	0	15	25
City 5	30	20	30	15	0	14
City 6	20	10	20	25	14	0

<u>Decision Variables</u>: $X_i = \begin{cases} 1 & \text{if a station is built in city } i \\ 0 & \text{otherwise.} \end{cases}$

<u>Objective Function</u>: Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

<u>Constraints</u>: Ensuring City 1 is within 15 miles of a station is equivalent to $X_1 + X_2 \geq 1$ since City 1 and City 2 are within 15 miles of each other. We apply this logic to each city.

(city 1) $X_1 + X_2 \geq 1$
(city 2) $X_1 + X_2 + X_6 \geq 1$
(city 3) $X_3 + X_4 \geq 1$
(city 4) $X_3 + X_4 + X_5 \geq 1$
(city 5) $X_4 + X_5 + X_6 \geq 1$
(city 6) $X_2 + X_5 + X_6 \geq 1$
$X_i \geq 0$

These two combined "cover" all rows.

Optimal Solution:
$X_2 = X_4 = 1$
$X_1 = X_3 = X_5 = X_6 = 0$
Min $Z = 2$

⟹ Build a station in City 2 and in City 4 and all other cities will be within 15 miles

2.] FIXED-CHARGE PROBLEM: I have been approached by three telephone companies to subscribe to their long-distance service in the United States. MaBell will charge a flat \$16 per month plus \$0.25 a minute. PaBell will charge \$25 per month but will reduce the per-minute cost to \$0.21. As for BabyBell, the flat monthly charge is \$18, and the cost per minute is \$0.22. I usually make an average of 200 minutes of long-distance calls a month. Assuming that I do not pay the flat monthly fee unless I make calls and that I can apportion my calls among all three companies, how should I use the three companies to minimize my monthly telephone bill?

Decision Variables:

Continuous (ℝ)
$\begin{cases} x_1 = \text{MaBell minutes used.} \\ x_2 = \text{PaBell minutes used.} \\ x_3 = \text{BabyBell minutes used.} \end{cases}$

Boolean ({0,1})
$\begin{cases} y_1 = \text{Subscribe to MaBell} \\ y_2 = \text{Subscribe to PaBell} \\ y_3 = \text{Subscribe to BabyBell} \end{cases}$

Objective Function:

Minimize $Z = .25 x_1 + .21 x_2 + .22 x_3 + $

$16 y_1 + 25 y_2 + 18 y_3$

Constraints:

(Max # of minutes) $x_1 + x_2 + x_3 \le 200$

(minutes are used only if subscribed)

$x_1 \le 200 y_1$
$x_2 \le 200 y_2$
$x_3 \le 200 y_3$

$y_1 \le 1$
$y_2 \le 1$
$y_3 \le 1$

$x_i, y_i \ge 0, \quad y_i = \text{integer}.$