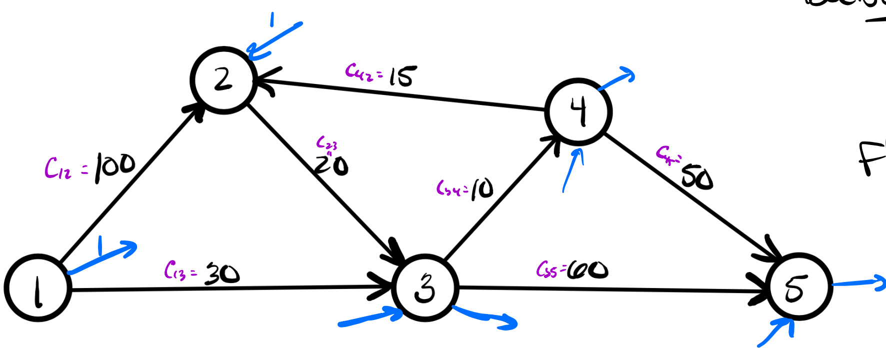
§8.5: Minimum Cost Network Flow Problems

1.] Consider the familiar network given below. Formulate the shortest path problem from node 1 to node 2 as a MCNFP.



Decision Vars: x_{ij} = amount of flow on edge (i,j); $x_{ij} \in \{0,1\}$

Flow Constraints:

(node 1) $x_{12} + x_{13} = 1$

(node 2) $x_{23} - x_{12} - x_{42} = -1$

(node 3) $x_{34} + x_{35} - x_{13} - x_{23} = 0$

(node 4) $x_{42} + x_{45} - x_{34} = 0$

(node 5) $-x_{35} - x_{45} = 0$

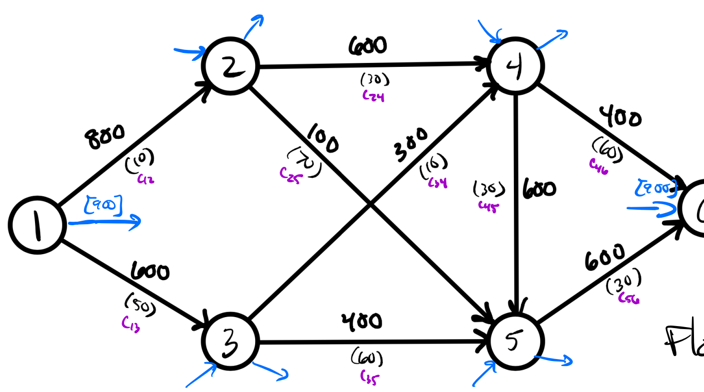
Obj. Function:

Minimize $Z = 100x_{12} + 30x_{13} + 20x_{23} + \cdots$
$10x_{34} + 60x_{35} + 15x_{42} + 50x_{45}$

Lower Bounds:

$x_{ij} \geq 0$

2.] Each hour, a total of 900 cars enter a network at node 1 and seek to travel to node 6. On the graph, the weight of each edge represents the maximum number of cars that can pass by any point on the arc during a one-hour period. The number in parentheses represents the time it takes a car to traverse that arc. Formulate the MCNFP that minimizes total time for all cars to travel from node 1 to node 6.



Decision Vars:

x_{ij} = # of cars travelling on edge (i,j) each hour.

Flow Constraints:

(node 1) $x_{12} + x_{13} = 900$

(node 2) $x_{24} + x_{25} - x_{12} = 0$

(node 3) $x_{34} + x_{35} - x_{13} = 0$

(node 4) $x_{45} + x_{46} - x_{24} - x_{34} = 0$

(node 5) $x_{56} - x_{35} - x_{25} - x_{45} = 0$

(node 6) $-x_{46} - x_{56} = -900$

Lower/Upper Bounds

$0 \leq x_{12} \leq 800$

$0 \leq x_{13} \leq 600$

$0 \leq x_{24} \leq 600$

$0 \leq x_{25} \leq 100$

$0 \leq x_{34} \leq 300$

$0 \leq x_{35} \leq 400$

$0 \leq x_{45} \leq 600$

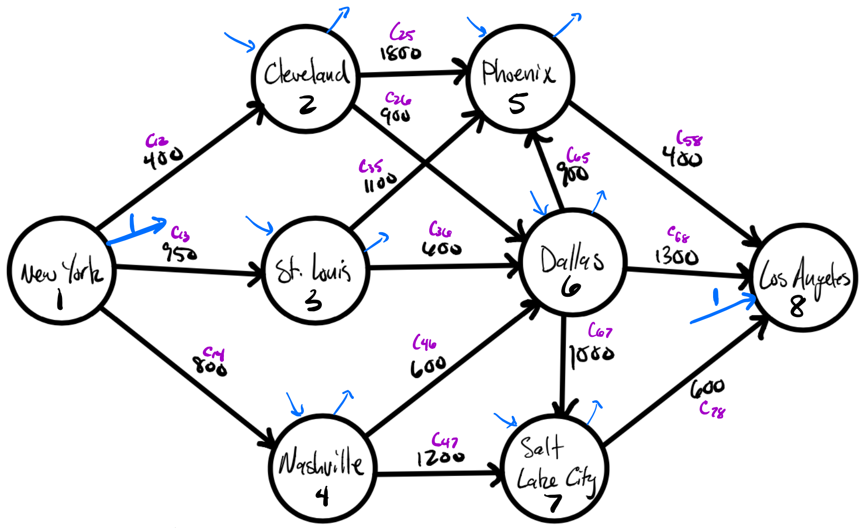
$0 \leq x_{46} \leq 400$

$0 \leq x_{56} \leq 600$

Objective Function:

Min $Z = 10x_{12} + 50x_{13} + 30x_{24} + 70x_{25} + 10x_{34} + \cdots$
$60x_{35} + 30x_{45} + 60x_{46} + 30x_{56}$

3.] A truck must travel from New York to Los Angeles. As shown in the graph below, a variety of routes are available. The number associated with each arc is the number of gallons of fuel required by the truck to traverse the arc. Formulate an MCNFP for finding the New York to Los Angeles route that uses the minimum amount of gas.



Decision Variables:

$x_{i,j}$ = amount of truck flow on edge (i,j); $x_{i,j} = \{0,1\}$

Objective Function:

$$\text{Min } z = \sum_{\text{all edges}} c_{ij} x_{ij}$$

Constraints:

$$x_{12} + x_{13} + x_{14} = 1$$

$$x_{25} + x_{26} - x_{12} = 0$$

$$x_{35} + x_{36} - x_{13} = 0$$

$$x_{46} + x_{47} - x_{14} = 0$$

$$x_{58} - x_{25} - x_{35} - x_{65} = 0$$

$$x_{65} + x_{67} + x_{68} - x_{26} - x_{36} - x_{46} = 0$$

$$x_{78} - x_{47} - x_{67} = 0$$

$$-x_{58} - x_{68} - x_{78} = -1$$

$$x_{ij} \geq 0.$$