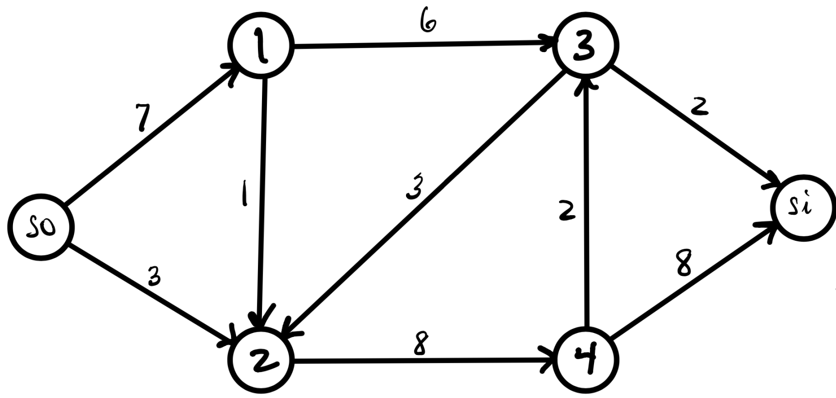
# §8.3: Maximum Flow Problems

1.] For the network given below, set up an LP that will find the maximum flow from source to sink.




Decision Variables

$x_{ij}$ = amount of flow from node $i$ to node $j$.

Objective Function:

Max $z = x_{so,1} + x_{so,2}$

or Max $z = x_{3,si} + x_{4,si}$

Constraints:

(Conservation)

$x_{so,1} = x_{13} + x_{12}$

$x_{so,2} + x_{12} + x_{32} = x_{24}$

$x_{13} + x_{43} = x_{32} + x_{3,si}$

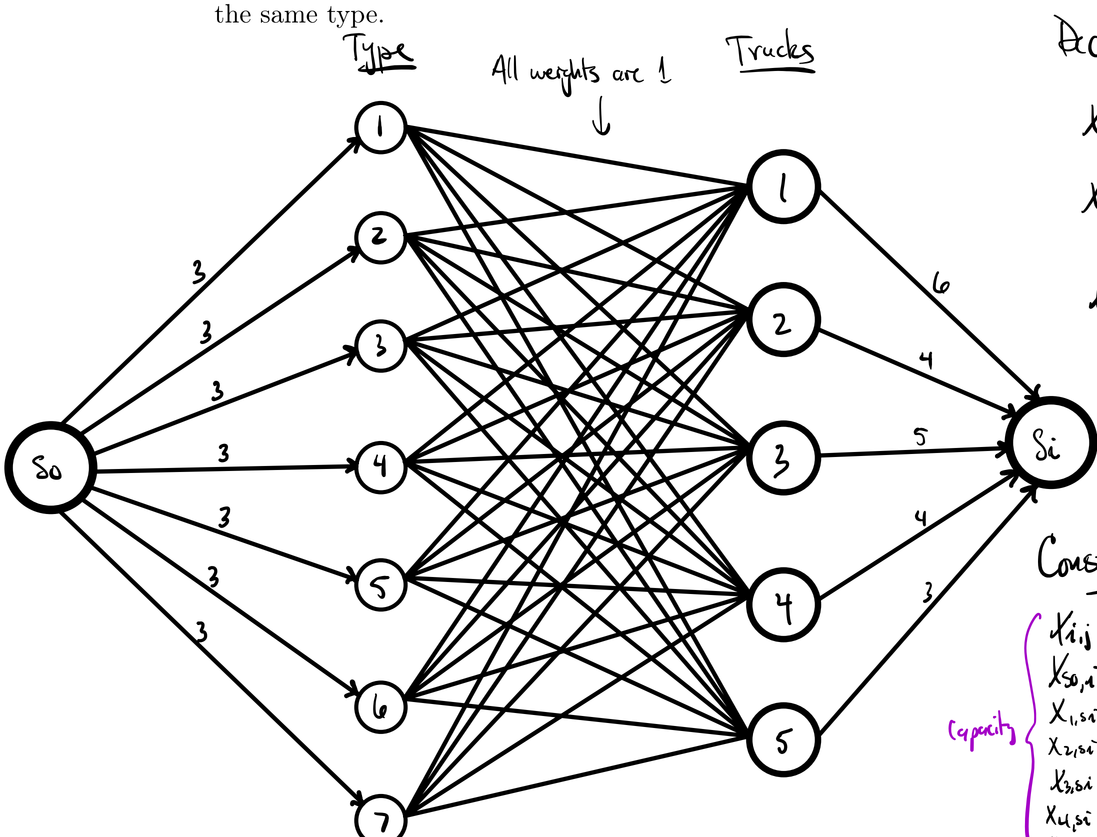
$x_{24} = x_{43} + x_{4,si}$

(Capacity)

$x_{so,1} \le 7$, $x_{so,2} \le 3$, $x_{12} \le 1$, $x_{13} \le 6$, $x_{24} \le 8$, $x_{32} \le 3$

$x_{3,si} \le 2$, $x_{4,si} \le 4$, $x_{42} \le 2$, $x_{ij} \ge 0$

2.] Seven types of packages are to be delivered by five trucks. There are three packages of each type, and the capacities of the five trucks are 6, 4, 5, 4, and 3, respectively. Set up a maximum flow problem that can be used to determine whether the packages can be loaded so that no truck carries two packages of the same type.




Decision Variables:

$x_{so,i}$ = # of packages of each type $i$ $(i=1,2,\dots,7)$

$x_{j,si}$ = # of distinct packages transported by truck $(j=1,2,\dots,5)$

$x_{i,j}$ = # of packages of type $i$ loaded into truck $j$ $(i=1,2,\dots,7;\ j=1,2,\dots,5)$

Objective Function:

Max $z = \sum_{j=1}^{5} x_{j,si}$

Constraints:

(Capacity)

$x_{i,j} \le 1$ for all $i,j$

$x_{so,i} \le 3$ for all $i$

$x_{1,si} \le 6$

$x_{2,si} \le 4$

$x_{3,si} \le 5$

$x_{4,si} \le 4$

$x_{5,si} \le 3$

Continuity

$x_{so,i} = \sum_{j=1}^{5} x_{i,j}$ for each $i$

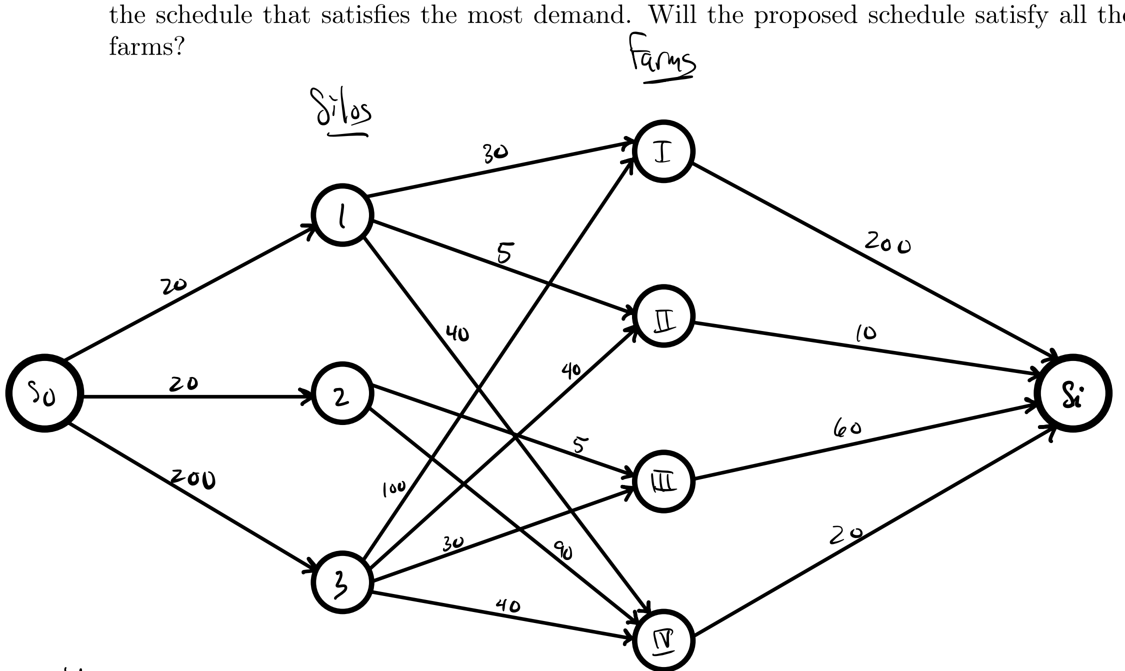
$\sum_{i=1}^{7} x_{i,j} = x_{j,si}$ for each $j$.

$x_{so,i},\ x_{j,si},\ x_{i,j} \ge 0$

3.] Chicken feed is transported by trucks from three silos to four farms. Some of the silos cannot ship directly to some of the farms. The capacities of the other routes are limited by the number of trucks available and the number of trips made daily. The following table shows the daily amounts of supply at the silos and the demand at the farms (in thousands of pounds). The cell entries of the table specify the daily capacities of the associated routes.

| | Farms | | | | |
|---|---|---|---|---|---|
| Silo | *I* | *II* | *III* | *IV* | **Supply** |
| 1 | 30 | 5 | 0 | 40 | **20** |
| 2 | 0 | 0 | 5 | 90 | **20** |
| 3 | 100 | 40 | 30 | 40 | **200** |
| **Demand** | **200** | **10** | **60** | **20** | |

Sketch a graph representation of this problem and formulate an LP to be solved in Excel to determine the schedule that satisfies the most demand. Will the proposed schedule satisfy all the demand at the farms?




Decision Variables:

$X_{so,i}$ = chicken feed supply at silo $i$ $(i=1,2,3)$

$X_{j,si}$ = chicken feed demand at farm $j$ $(j=1,2,3,4)$

$y_{i,j}$ = amount of chicken feed supplied from silo $i$ to farm $j$.

Objective function:

Max $Z = \sum_{i=1}^{3} X_{so,i}$

or Max $Z = \sum_{j=1}^{4} X_{j,so}$

Constraints:

$X_{so,1} \le 20$
$X_{so,2} \le 20$
$X_{so,3} \le 200$
$X_{1,si} \le 200$
$X_{2,si} \le 10$
$X_{3,si} \le 60$
$X_{4,si} \le 20$
$X_{so,i} \ge 0$
$X_{j,si} \ge 0$

$y_{11} \le 30$
$y_{12} \le 5$
$y_{14} \le 40$
$y_{23} \le 5$
$y_{24} \le 90$
$y_{31} \le 100$
$y_{32} \le 30$
$y_{33} \le 40$
$y_{34} \le 30$
$y_{i,j} \ge 0$

$X_{so,1} = y_{11} + y_{12} + y_{14}$
$X_{so,2} = y_{23} + y_{24}$
$X_{so,3} = y_{31} + y_{32} + y_{33} + y_{34}$
$y_{11} + y_{31} = X_{1,si}$
$y_{12} + y_{32} = X_{2,si}$
$y_{23} + y_{33} = X_{3,si}$
$y_{14} + y_{24} + y_{34} = X_{4,si}$
$y_{ij} \ge 0.$