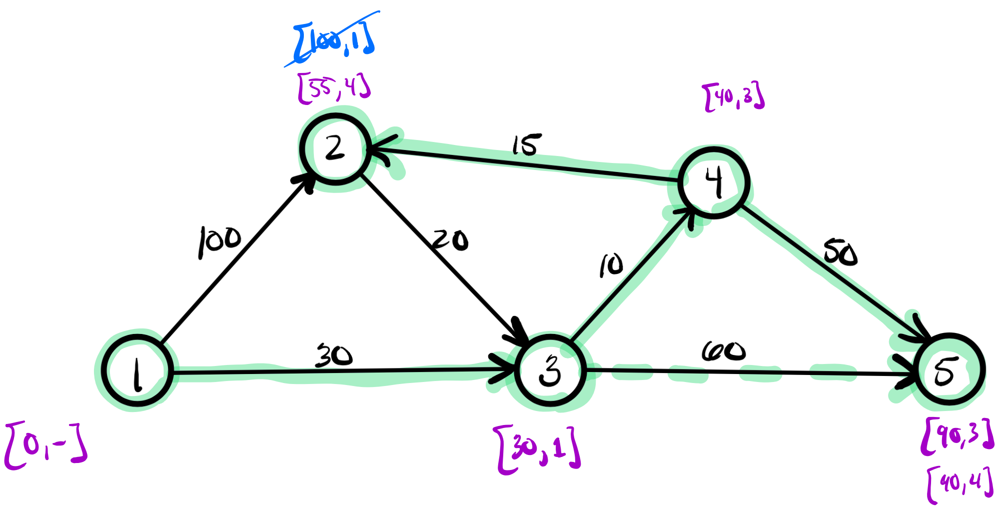
§8.2 (PART 2): DIJKSTRA'S ALGORITHM

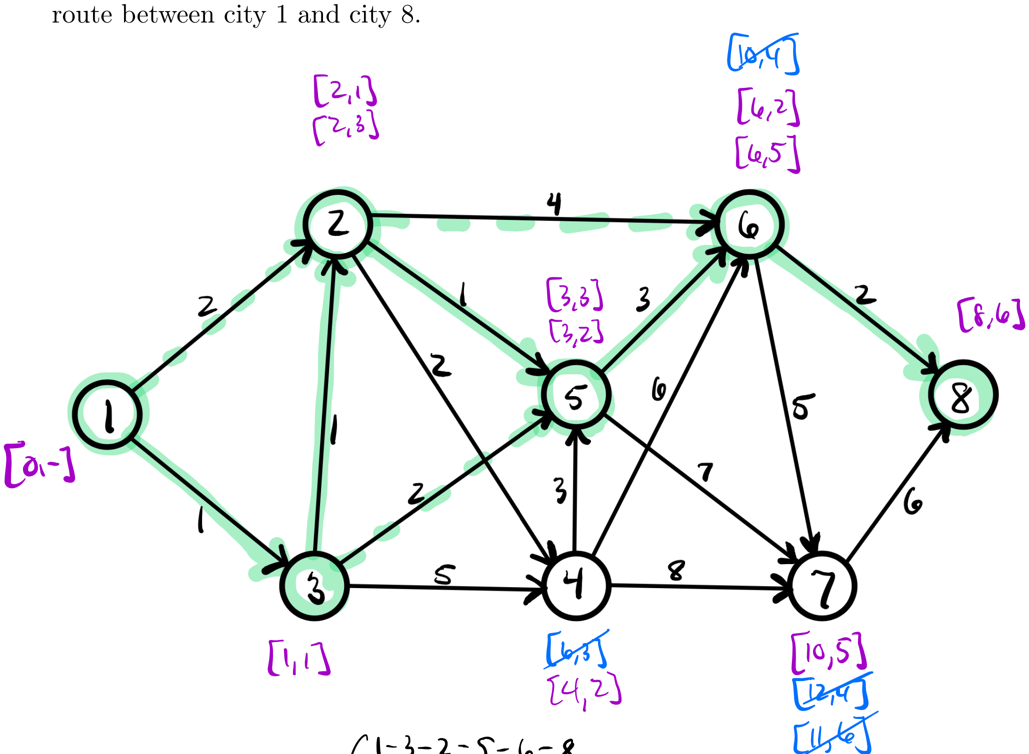
1.] Consider the network given below. The graph shows the permissible routes and their lengths in miles between city 1 (node 1) and four other cities (nodes 2 to 5). Use Dijkstra's algorithm to find the shortest route between city 1 and each of the remaining four cities.



Shortest Paths:

Source	Dest.	Path	Cost
1	3	1-3	30
1	4	1-3-4	40
1	2	1-3-4-2	55
1	5	{1-3-5, 1-3-4-5}	90

2.] Consider the network given below. The graph shows the permissible routes and their lengths in miles between city 1 (node 1) and seven other cities (nodes 2 to 8). Use Dijkstra's algorithm to find the shortest route between city 1 and city 8.



Shortest Paths:
1-3-2-5-6-8
1-2-5-6-8
1-3-2-4-8
1-3-5-6-8
1-2-6-8

Min Distance/Cost: 8

3.] DirectCo sells an item whose demands over the next 4 months are 100, 140, 210, and 180 units, respectively. The company can stock just enough supply to meet each month's demand, or it can overstock to meet the demand for two or more consecutive months. In the latter case, a holding cost of \$1.20 is charged per overstocked unit per month. DirectCo estimates the unit purchase prices for the next 4 months to be \$15, \$12, \$10, and \$14, respectively. A setup cost of \$200 is incurred each time a purchase order is placed. The company wants to develop a purchasing plan that will minimize the total costs of ordering, purchasing, and holding the item in stock. Formulate the problem as a shortest path model on an appropriate network. Then, solve the problem using Dijkstra's algorithm.

Nodes: i is the month in which an order is placed.

Edges: $i \to j$ cost of ordering stock to fill demand for intervals between months i and j. (e.g. $1 \xrightarrow{c} 4$ denotes the cost, c, for ordering stock will fill demand for months 1, 2, and 3)

Edge costs:

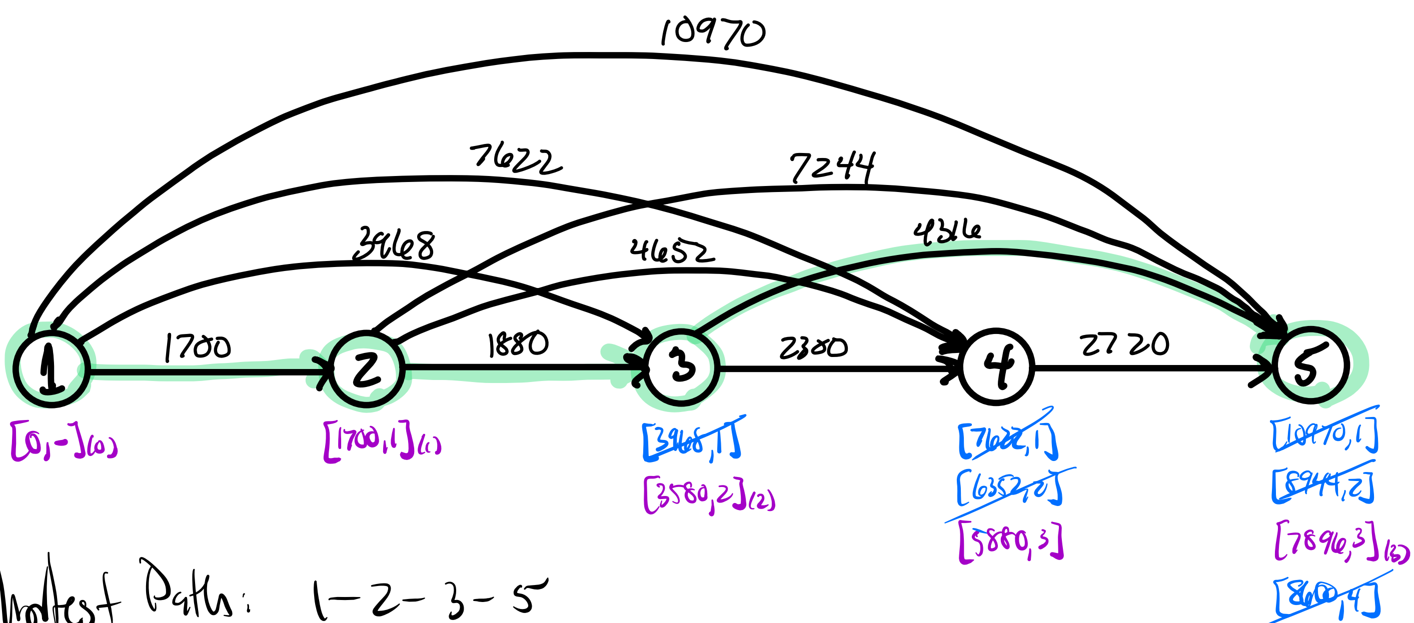
$1 \to 2$: $200 + 15 \cdot 100 = 1700$

$1 \to 3$: $200 + 15 \cdot 240 + 1.2 \cdot 140 = 3968$

$1 \to 4$: $200 + 15 \cdot 450 + 1.2 \cdot 140 + 2.4 \cdot 210 = 7622$

$1 \to 5$: $200 + 15 \cdot 630 + 1.2 \cdot 140 + 2.4 \cdot 210 + 3.6 \cdot 180 = 10970$

⋮



Shortest Path: 1 – 2 – 3 – 5

Solution: order month 1 stock in month 1, order month 2 stock in month 2, order months 3 & 4 stock in month 3.