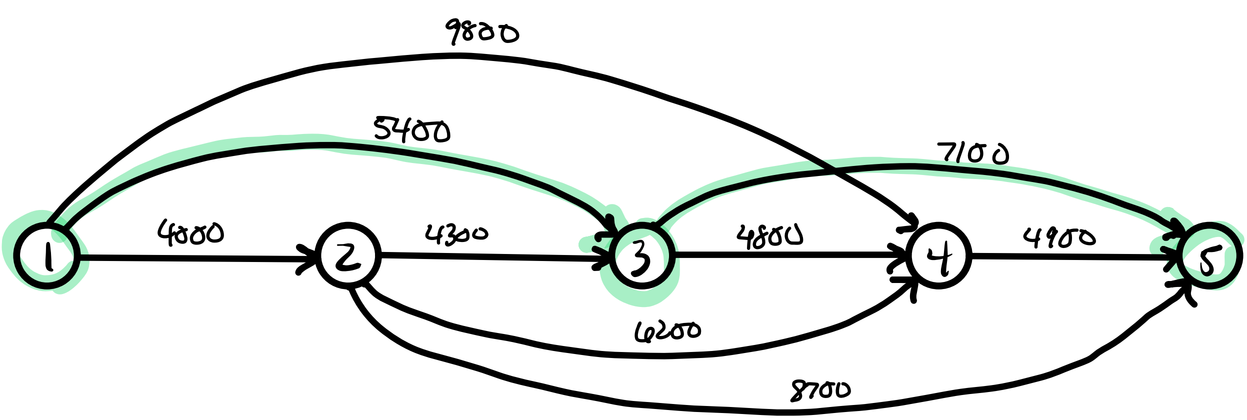
§8.2 (PART 1): SHORTEST PATH PROBLEM

1.] RentCar is developing a replacement policy for its car fleet over a 4-year planning horizon. At the start of each year, a car is either replaced or kept in operation for an extra year. A car must be in service for 1 to 3 years. The following table provides the replacement costs as a function of the year a car is acquired and the number of years in operation.

	Replacement cost ($) for given years in operation		
Equipment acquired	*1*	*2*	*3*
in start of year 1	4000	5400	9800
in start of year 2	4300	6200	8700
in start of year 3	4800	7100	–
in start of year 4	4900	–	–

Formulate this problem as a network and determine the shortest path from start of year 1 to the start of year 5 by inspection.

- Let each node i represent the start of year i $(i=1,2,\dots,5)$
- Let each edge (i,j) represent the replacement cost of replacing a car acquired in year i that has been in operation until start of year j $(j>i$, operated for $j-i$ years, $1\le j-i\le 3)$

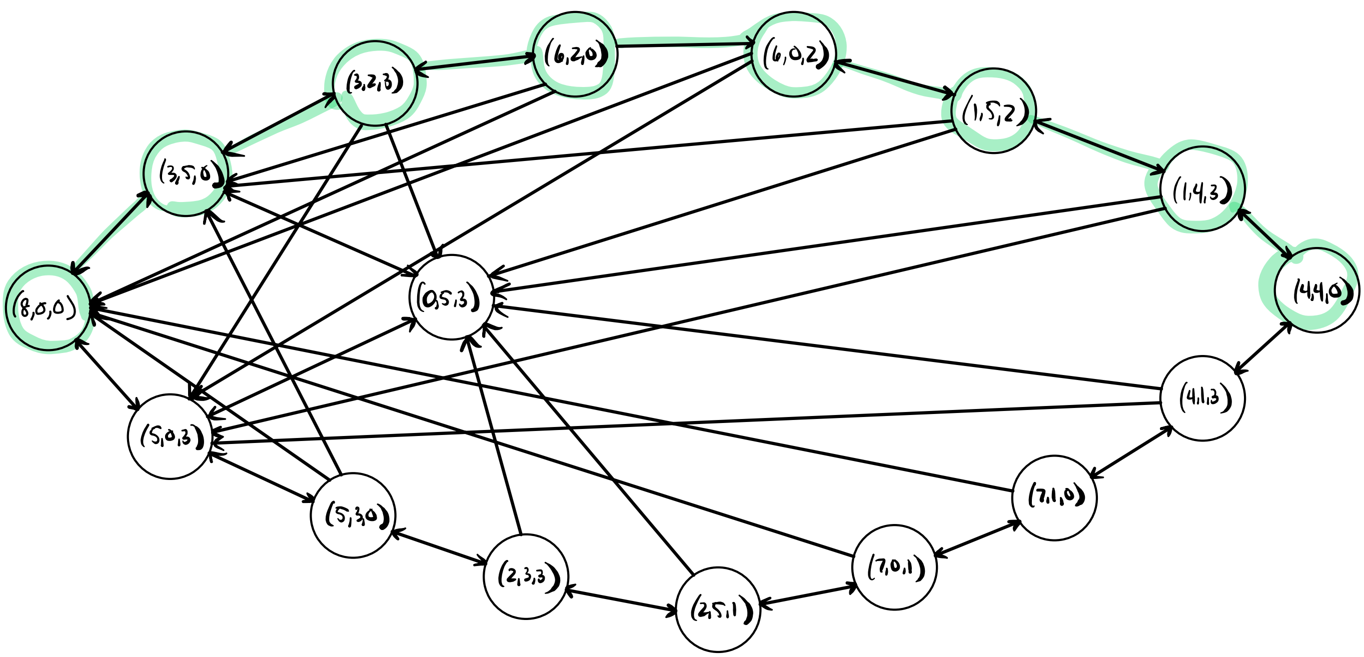


Shortest Path: $\{(1,3),(3,5)\}$ Min Cost: $12,500

Any car acquired in year 1 should be in operation until the start of year 3, then the replacement car is in operation until start of year 5.

2.] An 8-gallon jug is filled with fluid. Given two empty 5- and 3-gallon jugs, divide the 8 gallons of fluid into two equal parts using only the three jugs. What is the smallest number of transfers (decantations) needed to achieve this goal? Define nodes and edges to formulate this problem as a network. Then, solve the shortest path problem by inspection.

- Let node (i,j,k) denote the state of each jug where there are i gallons in the 8-gal jug, j gallons in the 5-gal jug, and k gallons in the 3-gal jug.
- Let an edge represent a transfer of liquid so that a single jug is either emptied or filled.



Shortest Path: Highlighted in green above

7 steps.