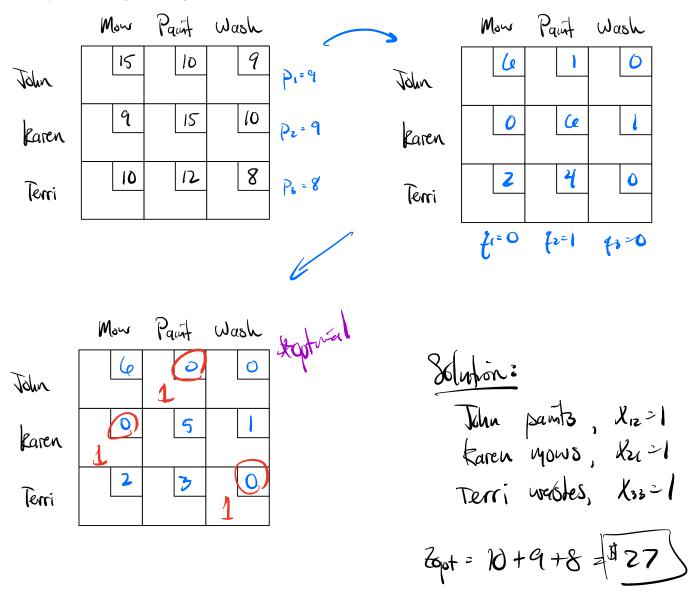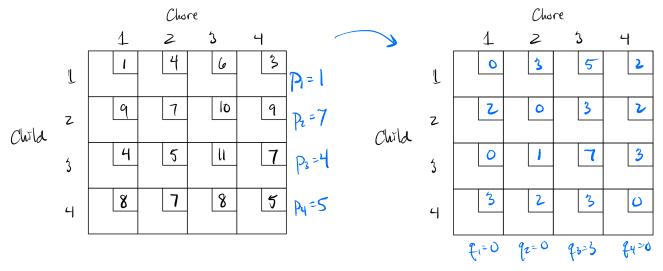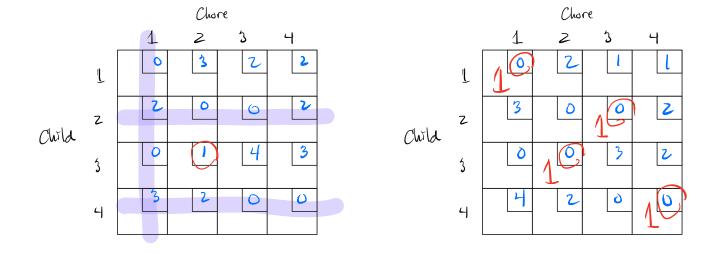## §7.5: ASSIGNMENT PROBLEMS

1.] Joe Klyne's three children, John, Karen, and Terri, want to earn some money for personal expenses. Mr. Klyne has chosen three chores for this children: mowing the lawn, painting the garage door, and washing the family cars. To avoid anticipated sibling competition, he asks them to submit individual (secret) bids for what they feel is fair pay pay for each of the three chores. The table below summarizes the bids. The children will abide by their father's decision regarding the assignment of chores. Solve the assignment problem using the Hungarian method.

	Mow	Paint	Wash
John	15	10	9
Karen	9	15	10
Terri	10	12	8

$p_1 = 9$
$p_2 = 9$
$p_3 = 8$

	Mow	Paint	Wash
John	6	1	0
Karen	0	6	1
Terri	2	4	0

$q_1 = 0 \quad q_2 = 1 \quad q_3 = 0$

	Mow	Paint	Wash
John	6	⓪ 1	0
Karen	⓪ 1	5	1
Terri	2	3	⓪ 1

*optimal

Solution:

John paints, $x_{12} = 1$
Karen mows, $x_{21} = 1$
Terri washes, $x_{33} = 1$

$z_{opt} = 10 + 9 + 8 = \boxed{\$27}$

2.] Suppose that the situation in the previous problem was extended to four children and four chores with costs summarized in the grid tableau below.

Chore

Child	1	2	3	4	
1	1	4	6	3	$P_1 = 1$
2	9	7	10	9	$P_2 = 7$
3	4	5	11	7	$P_3 = 4$
4	8	7	8	5	$P_4 = 5$

Chore

Child	1	2	3	4
1	0	3	5	2
2	2	0	3	2
3	0	1	7	3
4	3	2	3	0

$q_1 = 0 \quad q_2 = 0 \quad q_3 = 3 \quad q_4 = 0$

Chore

Child	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	(1)	4	3
4	3	2	0	0

Chore

Child	1	2	3	4
1	(0)	2	1	1
2	3	0	(0)	2
3	0	(0)	3	2
4	4	2	0	(0)

Solution: $X_{11} = 1$, $X_{32} = 1$, $X_{13} = 1$, $X_{44} = 1$

$Z_{opt} = 1 + 5 + 10 + 5 = 21$