§7.4: Sensitivity Analysis in Transportation Problems

1.] Consider the optimal solution for the Transportation LP below:

	$v_1=-3$	$v_2=2$	$v_3=4$	$v_4=11$	Supply
$u_1=0$	cost 10	cost 2; 5	cost 20	cost 11; 10	15
$u_2=5$	cost 12	cost 7; 10	cost 9; 15	cost 20	25
$u_3=7$	cost 4; 5	cost 14	cost 16	cost 18; 5	10
Demand	5	15	15	15	

$Z_{opt} = 435$

a.) Determine the range of values of c_{33} for which the current basis remains optimal.

New $C_{33} = 16 + \Delta C_{33}$ $\Rightarrow$ $\bar{C}_{33} = u_3 + v_3 - c_{33} \le 0$

$\Rightarrow$ $7 + 4 - (16 + \Delta C_{33}) \le 0$

$\Rightarrow$ $-5 - \Delta C_{33} \le 0$ $\Rightarrow$ $\boxed{\Delta C_{33} \ge -5}$

b.) Determine the range of values of c_{12} for which the current basis remains optimal.

New $C_{12} = 2 + \Delta C_{12}$

Basic Vars:

$u_1 + v_2 = 2 + \Delta C_{12}$
$u_1 + v_4 = 11$
$u_2 + v_2 = 7$
$u_2 + v_3 = 9$
$u_3 + v_1 = 4$
$u_3 + v_4 = 18$

$\Rightarrow$

$u_1 = 0$
$v_4 = 11$
$v_2 = 2 + \Delta C_{12}$
$u_2 = 5 - \Delta C_{12}$
$v_3 = 4 + \Delta C_{12}$
$u_3 = 7$
$v_1 = -3$

Non basic Vars:

$\bar{C}_{11} = u_1 + v_1 - C_{11} = -13$
$\bar{C}_{13} = u_1 + v_3 - C_{13} = \Delta C_{12} - 16 \le 0$
$\bar{C}_{21} = u_2 + v_1 - C_{21} = -\Delta C_{12} - 10 \le 0$
$\bar{C}_{24} = u_2 + v_4 - C_{24} = -\Delta C_{12} - 4 \le 0$
$\bar{C}_{32} = u_3 + v_2 - C_{32} = \Delta C_{12} - 5 \le 0$
$\bar{C}_{33} = u_3 + v_3 - C_{33} = \Delta C_{12} - 5 \le 0$

$$\Rightarrow \begin{cases} \Delta C_{12} \le 16 \\ \Delta C_{12} \ge -10 \\ \Delta C_{12} \ge -4 \\ \Delta C_{12} \le 5 \\ \Delta C_{12} \le 5 \end{cases} \Rightarrow \boxed{-4 \le \Delta C_{12} \le 5}$$

c.) If s_2 and d_2 are both increased by 2, what is the new optimal solution?

	$V_1=-3$	$V_2=2$	$V_3=4$	$V_4=11$	Supply
$U_1=0$	10	2; 5	20	11; 10	15
$U_2=5$	12	7; ~~10~~ 12	9; 15	20	~~25~~ 27
$U_3=7$	4; 5	14	16	18; 5	10
Demand	5	~~15~~ 17	15	15	

Since x_{22} is a basic variable, we simply increase x_{22} by 2.

Hence, $x_{22}=12$, the other variables remain the same.

New $z_{opt} = 435 + 2(5) + 2(2) = \boxed{449}$

d.) If s_3 and d_3 are both increased by 3, what is the new optimal solution?

	$V_1=-3$	$V_2=2$	$V_3=4$	$V_4=11$	Supply
$U_1=0$	10	2; 5+3	20	11; 10−3	15
$U_2=5$	12	7; 10−3	9; 15+3	20	25
$U_3=7$	4; 5	14	16	18; 5+3	~~10~~ 13
Demand	5	15	~~15~~ 18	15	

Since x_{33} is not a basic variable, we create a loop with cell x_{33} involved. Any basic variable in the loop is appropriately increased/decreased.

New solution: $x_{12}=8$, $x_{14}=7$, $x_{22}=7$, $x_{23}=18$, $x_{31}=5$, $x_{34}=8$

$z_{opt} = 8(2) + 7(11) + 7(7) + 18(9) + 5(4) + 8(18) = \boxed{468}$