§6.9: Duality and Sensitivity Analysis for Objective Function Coefficients and New Activities

1.] SugarCo manufactures three types of candy bar: Whatchamacallit, Peanut Chews, and Abba-Zabba. Each bar consists totally of sugar and chocolate. The compositions of each type of candy bar and the profit earned from each candy bar are in the table below. Fifty oz of sugar and 100 oz of chocolate are available. To maximize profits, SugarCo formulates the following LP:

Candy Bar	Sugar (ounces)	Chocolate (ounces)	Profit (cents)
Whatchamacallit	1.5	2.75	3
Peanut Chews	1	3	7
Abba-Zabba	1	1	5

Maximize: $z = 3x_1 + 7x_2 + 5x_3$

Subject to: $x_1 + x_2 + x_3 \leq 50$

$2x_1 + 3x_2 + x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

The optimal tableau is

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	RHS
0	z	1	3	0	0	4	1	300
1	x_3	0	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	25
2	x_2	0	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	25

a.) For what values of profit on the Whatchamacallit does the current basis remain optimal?

- Change 3 to c_1.
- First constraint of the dual: $y_1 + 2y_2 \geq c_1$

  Plug in dual sol $y_1 = 4, y_2 = 1$ → $4 + 2(1) \geq c_1$

  $6 \geq c_1$
- Whatchamacallit can make a max profit of 6 cents under the current basis.

b.) If a Whatchamacallit used 0.5 oz of sugar and 0.75 oz of chocolate, would the current basis remain optimal?

Here, $a_{11} = .5$ and $a_{21} = .75$.

$$a_{11}y_1 + a_{21}y_2 \geq 3$$

$$(.5)(4) + (.75)(1) \geq 3$$

$$2.75 \not\geq 3$$

x_2, x_3 is no longer an optimal basis.

c.) SugarCo is considering making an Idaho Spud that yields \$0.10 profit and uses 2 oz of sugar and 1 oz of chocolate. Does the current basis remain optimal?

Adding the following: $c_4 = 10$ $a_{14} = 2$ $a_{24} = 1$

c_4: Profit for Idaho Spud

a_{14}: Sugar Content of Idaho Spud

a_{24}: Choc Content of Idaho Spud

$$a_{14}y_1 + a_{24}y_2 \geq c_4$$

$$(2)(4) + (1)(1) \overset{?}{\geq} 10$$

$$9 \not\geq 10$$

Basis would change. It is worth it to make Spuds from Idaho because the cost to produce it (9 cents) is less than the profit made (10 cents).