§6.8: Shadow Prices (Feasibility Ranges)

1.] A company manufactures two products (1 and 2). Each unit of product 1 can be sold for \$15, and each unit of product 2 can be sold for \$25. Each product requires raw material and two types of labor (skilled and unskilled) (see the table below). Currently, the company has available 100 hours of skilled labor, 70 hours of unskilled labor, and 30 units of raw material. Because of marketing considerations, at least 3 units of product 2 must be produced. The relevant LP is given below.

	Product	
Resource	1	2
Skilled Labor (hours)	3	4
Unskilled Labor (hours)	2	3
Raw Material (units)	1	2

$$\begin{aligned} \text{Maximize:} \quad & z = 15x_1 + 25x_2 \\ \text{Subject to:} \quad & 3x_1 + 4x_2 \leq 100 \\ & 2x_1 + 3x_2 \leq 70 \\ & x_1 + 2x_2 \leq 30 \\ & x_2 \geq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Suppose Row 0 of the optimal tableau is

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	e_4	a_4	RHS
0	z	1	0	0	0	0	15	5	$(M-5)$	435

a.) How much would the company be willing to pay for an extra unit of raw material?

b.) Assuming the current basis remains optimal, what would the company's revenue be if 35 units of raw material were available?

c.) Assuming the current basis remains optimal, what would the company's revenue be if 80 hours of skilled labor were available?

d.) Assuming the current basis remains optimal, what would the company's revenue be if at least 5 units of product 2 were required?

2.] The ToyCo company uses three operations to assemble three types of toys – trains, trucks, and cars. The daily available times for the three operations are 430, 460, and 420 minutes, respectively, and the revenues per unit of toy train, truck, and car are \$3, \$2, and \$5, respectively. The assembly times per train at the three operations are 1, 3, and 1 minutes, respectively. The corresponding times per train and per car are $(2, 0, 4)$ and $(1, 2, 0)$, respectively. Letting x_1, x_2, and x_3 be the daily number of units assembled of trains, trucks, and cars, respectively, the associated LP and optimal tableau are given as:

$$\begin{aligned} \text{Maximize:} \quad & z = 3x_1 + 2x_2 + 5x_3 \\ \text{Subject to:} \quad & x_1 + 2x_2 + x_3 \leq 430 \\ & 3x_1 \quad + 2x_3 \leq 460 \\ & x_1 + 4x_2 \quad \leq 420 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0	z	1	4	0	0	1	2	0	1350
1	x_2	0	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
2	x_3	0	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
3	s_3	0	2	0	0	-2	1	1	20

a.) What is the feasibility range of operation 1?

b.) Suppose the availabilities of operations 1, 2, and 3 are changed to 438, 500, and 410 minutes, respectively. Use the set of simultaneous conditions to show that the current basis remains feasible, and determine the new optimal value using the shadow prices.