## $\S 6.7$ (Part 1): The Duality Theorem

1.] Prove: If the primal is unbounded, then the dual problem is infeasible.
2.] For the following LP,

$$
\begin{aligned}
& \text { Maximize: } \quad z=-x_{1}+5 x_{2} \\
& \text { Subject to: } \quad x_{1}+2 x_{2} \leq 0.5 \\
& -x_{1}+3 x_{2} \leq 0.5 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

the Row 0 of the optimal tableau is

| Row | Basic | $z$ | $x_{1}$ | $x_{2}$ | $s_{1}$ | $s_{2}$ | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $z$ | 1 | 0 | 0 | 0.4 | 1.4 | $? ?$ |

What is the optimal $z$-value of the given LP?
3.] Consider the following LP:

$$
\begin{aligned}
\text { Maximize: } & z=-2 x_{1}-x_{2}+x_{3} \\
\text { Subject to: } \quad x_{1}+x_{2}+x_{3} & \leq 3 \\
x_{2}+x_{3} & \geq 2 \\
x_{1}+x_{3} & =1 \\
x_{1}, x_{2}, x_{3} & \geq 0
\end{aligned}
$$

The Row 0 of the optimal tableau is

| Row | Basic | $z$ | $x_{1}$ | $x_{2}$ | $x_{3}$ | $s_{1}$ | $e_{2}$ | $a_{2}$ | $a_{3}$ | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $z$ | 1 | 4 | 0 | 0 | 0 | 1 | $(M-1)$ | $(M+2)$ | 0 |

What is the optimal solution to the dual LP? Verify that the optimal objective value function for the dual is the same as the primal.

