§6.6: Economic Interpretation of the Dual Problem

1.] Recall the Reddy Mikks problem, where a company produces both interior and exterior paints from two raw materials, M_1 and M_2. The first two constraints are raw material constraints. The third constraint makes sure the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Lastly, the maximum daily demand for interior paint is 2 tons. The objective function is measured in $1000s. The maximization LP is formulated as follows:

Primal LP: Dual LP:

$$\text{Maximize:} \quad z = 5x_1 + 4x_2$$

$$
\begin{aligned}
\text{Subject to:} \quad 6x_1 + 4x_2 &\leq 24 \\
x_1 + 2x_2 &\leq 6 \\
-x_1 + x_2 &\leq 1 \\
x_2 &\leq 2 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

a.) Find the dual problem and interpret the dual variables below.

b.) Note that $x_1 = x_2 = 1$ is a feasible solution for the primal and that $y_1 = y_2 = y_3 = y_4 = 1$ is a feasible solution for the dual. Show that $z < w$.

c.) Interpret the expression $\sum\limits_{i=1}^{m} a_{ij} y_i - c_j$.

d.) The optimal tableau for the Reddy Mikks problem is given below. Identify the optimal solution for the primal and deduce the optimal solution of the dual from this tableau. Then, show $z = w$ at optimality.

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
0	z	1	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21
1	x_1	0	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3
2	x_2	0	0	1	$-\frac{1}{8}$	$\frac{3}{4}$	0	0	$\frac{3}{2}$
3	s_3	0	0	0	$\frac{3}{8}$	$-\frac{5}{4}$	1	0	$\frac{5}{2}$
4	s_4	0	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2}$