# §6.5: Finding the Dual of an LP

1.] Find the dual of the following normal maximization LP:

$$\begin{aligned} \text{Maximize:} \quad & z = 2x_1 + x_2 \\ \text{Subject to:} \quad & -x_1 + x_2 \leq 1 \\ & x_1 + x_2 \leq 3 \\ & x_1 - 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Dual LP:

Min $w = y_1 + 3y_2 + 4y_3$

Subject to

$$\begin{aligned} -y_1 + y_2 + y_3 &\geq 2 \\ y_1 + y_2 - 2y_3 &\geq 1 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

2.] Find the dual of the following normal minimization LP:

$$\begin{aligned} \text{Minimize:} \quad & z = x_1 - x_2 \\ \text{Subject to:} \quad & 2x_1 + x_2 \geq 4 \\ & x_1 + x_2 \geq 1 \\ & x_1 + 2x_2 \geq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Dual LP

Max $w = 4y_1 + y_2 + 3y_3$

Subject to

$$\begin{aligned} 2y_1 + y_2 + y_3 &\leq 1 \\ y_1 + y_2 + 2y_3 &\leq -1 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

*Note, the dual LP is clearly infeasible (constraint 2 cannot be satisfied)

3.] Find the dual of the following non-normal minimization LP:

$$\begin{aligned} \text{Minimize:} \quad & z = 4x_1 + x_2 \\ \text{Subject to:} \quad & 3x_1 + x_2 = 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 4 \\ & x_1 \text{ urs}, x_2 \geq 0 \end{aligned}$$

- Convert every constraint to an equality constraint by defining $x_1 = x_1' - x_1''$ for the urs variable and adding an excess variable $x_3$ for constraint 2 and a slack variable $x_4$ for constraint 3.

**Primal LP:**

Min $z = 4x_1' - 4x_1'' + x_2 + 0x_3 + 0x_4$

Subject to

$$\begin{aligned} 3x_1' - 3x_1'' + x_2 &= 3 \\ 4x_1' - 4x_1'' + 3x_2 - x_3 &= 6 \\ x_1' - 4x_1'' + 2x_2 + x_4 &= 4 \\ x_1', x_1'', x_2, x_3, x_4 &\geq 0 \end{aligned}$$

- Now, since the Primal LP is a min problem, we set up the max dual LP with "$\leq$" constraints

**Dual LP:**

Max $w = 3y_1 + 6y_2 + 4y_3$

Subject to

$$\begin{aligned} 3y_1 + 4y_2 + y_3 &\leq 4 \\ -3y_1 - 4y_2 - y_3 &\leq -4 \\ y_1 + 3y_2 + 2y_3 &\leq 1 \\ -y_2 &\leq 0 \\ y_3 &\leq 0 \end{aligned}$$

Note: these two constraints collapse into a single equality constraint

$$\begin{cases} 3y_1 + 4y_2 + y_3 \leq 4 \\ -3y_1 - 4y_2 - y_3 \leq -4 \end{cases}$$

$$\Rightarrow \begin{cases} 3y_1 + 4y_2 + y_3 \leq 4 \\ 3y_1 + 4y_2 + y_3 \geq 4 \end{cases}$$

$$\Rightarrow 3y_1 + 4y_2 + y_3 = 4$$

**Dual LP:**

Max $w = 3y_1 + 6y_2 + 4y_3$

Subject to

$$\begin{aligned} 3y_1 + 4y_2 + y_3 &= 4 \\ y_1 + 3y_2 + 2y_3 &\leq 1 \\ y_1 \text{ urs}, y_2 \geq 0, y_3 &\leq 0 \end{aligned}$$

Note: the sign constraint on $y_1$ is urs because there are no inequalities saying otherwise.