§6.5: Intro to the Dual Problem
1.] Dakota Furniture Company: A furniture company manufactures desks, tables, and chairs. The manufacture of each type of furniture requires lumber and two types of skilled labor: finishing and carpentry. The amount of each resource needed to create each product, the selling price of each product, and the available amounts of each resource are provided in the table below:

|  | Product |  |  | Available |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Resource | Desk | Table | Chair | Resources |  |
| Lumber (board ft) | 8 | 6 | 1 | 48 |  |
| Finishing (hours) | 4 | 2 | 1.5 | 20 |  |
| Carpentry (hours) | 2 | 1.5 | 0.5 | 8 |  |
| Selling Price (\$) | 60 | 30 | 20 |  |  |

a.) Your objective is to maximize revenue for the Dakota Furniture company. Formulate the maximization LP.
Define $x_{i}=\#$ of product $i$ sold.

$$
\begin{aligned}
& \max z=60 x_{1}+30 x_{2}+20 x_{3} \\
& \text { Subject to } 8 x_{1}+6 x_{2}+x_{3}
\end{aligned} \leq 480 子 \begin{aligned}
4 x_{1}+2 x_{2}+1.5 x_{3} & \leq 20 \\
2 x_{1}+1.5 x_{2}+0.5 x_{2} & \leq 8 \\
x_{1}, x_{2}, x_{3} & \geq 0
\end{aligned}
$$

b.) You're an entrepreneur willing to purchase all of Dakota Furniture company's resources. Formulate a minimization LP for the minimum purchasing price $w$ that you'll offer to Dakota after considering the value of each unit of resource. Define $y_{i}$ as the price paid for one unit of resource $i$.
Defwe $y_{i}=$ erie pard for 1 unit of resource $i$ (hard feet, fwishny hours, carpentry hours)

$$
\begin{aligned}
& \text { Min } \omega=48 y_{1}+20 y_{2}+8 y_{2} \\
& \text { Subject to } \quad 8 y_{1}+4 y_{2}+2 y_{3} \geq 60 \\
& 6 y_{1}+2 y_{2}+1.5 y_{2} \geq 30 \\
& y_{1}+1.5 y_{2}+0.5 y_{3} \geq 20 \\
& y_{1}, y_{2}, y_{3} \geq 0
\end{aligned}
$$

