§6.10: Complementary Slackness

1.] Consider the following LP:

$$\text{Maximize:} \quad z = 5x_1 + 3x_2 + x_3$$

$$\begin{aligned} \text{Subject to:} \quad 2x_1 + x_2 + x_3 &\le 6 \\ x_1 + 2x_2 + x_3 &\le 7 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

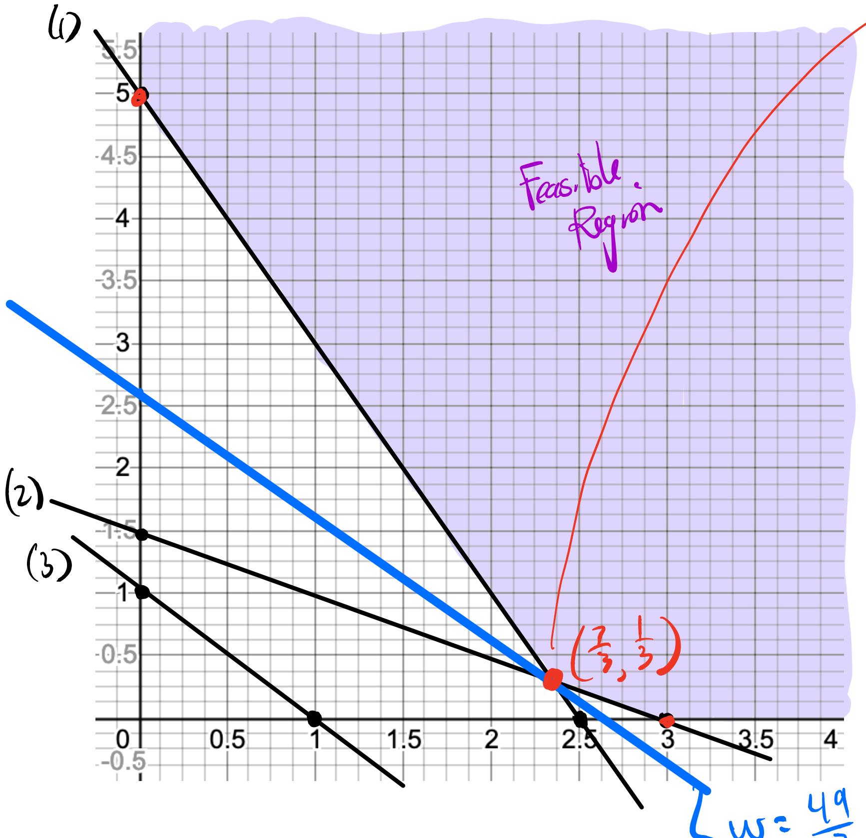
Graphically solve the dual of this LP. Then use complementary slackness to solve the max primal problem.

Dual LP:

Minimize $w = 6y_1 + 7y_2 \Rightarrow y = -\frac{6}{7}x + \frac{w}{7}$

Subject to
$2y_1 + y_2 \ge 5 \Rightarrow y \ge 5 - 2x \quad (1)$
$y_1 + 2y_2 \ge 3 \Rightarrow y \ge \frac{3}{2} - \frac{1}{2}x \quad (2)$
$y_1 + y_2 \ge 1 \Rightarrow y \ge 1 - x \quad (3)$
$y_1, y_2 \ge 0$

Optimal Solution:
$y_1 = 7/3 \quad y_2 = 1/3$
$w_{opt} = 49/3$



Dual (1) and (2) are binding, Dual (3) is redundant:

$e_1 = 0, \; e_2 = 0, \; e_3 > 0$

Complementary Slackness

$e_1 = 0 \Rightarrow x_1 > 0 \qquad y_1 > 0 \Rightarrow s_1 = 0$
$e_2 = 0 \Rightarrow x_2 > 0 \qquad y_2 > 0 \Rightarrow s_2 = 0$
$e_3 > 0 \Rightarrow x_3 = 0 \qquad$ (Primal (1) & (2) binding)

$$\begin{cases} 2x_1 + x_2 = 6 \\ x_1 + 2x_2 = 7 \end{cases} \Rightarrow \begin{cases} 2x_1 + x_2 = 6 \\ 2x_1 + 4x_2 = 14 \end{cases}$$

$$\Rightarrow \begin{cases} 2x_1 + x_2 = 6 \\ -3x_2 = -8 \end{cases} \Rightarrow \begin{cases} x_1 = 5/3 \\ x_2 = 8/3 \end{cases}$$

$$\boxed{\text{Primal sol: } x_1 = 5/3, \; x_2 = 8/3 \quad z_{opt} = 49/3}$$