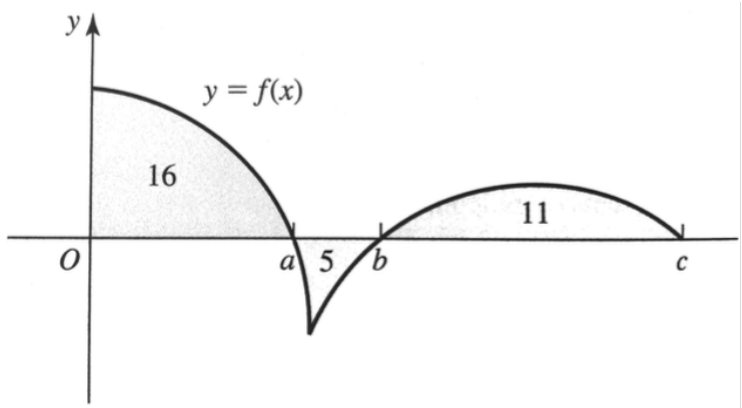
§5.2: The Definite Integral

1.] Consider the following function below with the areas of each region given.



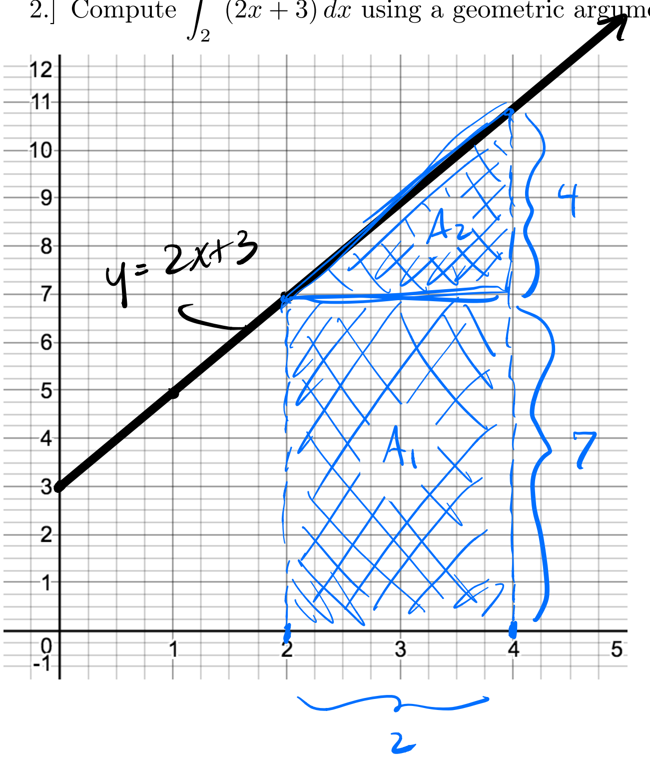
a.) $\int_0^a f(x)\,dx = \boxed{16}$

b.) $\int_0^b f(x)\,dx = 16+(-5) = \boxed{11}$

c.) $\int_a^c f(x)\,dx = (-5)+11 = \boxed{6}$

d.) $\int_0^c f(x)\,dx = 16+(5)+11 = \boxed{22}$

2.] Compute $\int_2^4 (2x+3)\,dx$ using a geometric argument.



$$\int_2^4 (2x+3)\,dx = A_1 + A_2$$

$$= lw + \tfrac{1}{2}bh$$

$$= (7)(2) + \tfrac{1}{2}(2)(4)$$

$$= 14 + 4$$

$$= \boxed{18}$$

3.] Suppose we only know the fact that $\int_0^4 3x(4-x)\,dx = 32$. Use properties of the definite integral to compute the following, if possible:

a.) $\int_4^0 3x(4-x)\,dx = -\int_0^4 3x(4-x)dx = \boxed{-32}$

b.) $\int_0^4 6x(4-x)\,dx = \int_0^4 2\cdot 3x(4-x)dx = 2\int_0^4 3x(4-x)dx = 2(32) = \boxed{64}$

c.) $\int_0^4 x(x-4)\,dx = -\int_0^4 x(4-x)dx = -\frac{1}{3}\int_0^4 3x(4-x)dx = -\frac{1}{3}(32) = \boxed{-\frac{32}{3}}$

d.) $\int_0^8 3x(4-x)\,dx = \int_0^4 3x(4-x)dx + \underbrace{\int_4^8 3x(4-x)dx}_{\text{unknown}} = 32 + \text{IDK} = \boxed{\text{Cannot be computed}}$

4.] Suppose we only know the fact that $\int_1^4 f(x)\,dx = 8$ and $\int_1^6 f(x)\,dx = 5$. Use properties of the definite integral to compute the following, if possible:

a.) $\int_1^4 (-3f(x))\,dx = -3\int_1^4 f(x)dx = -3(8) = \boxed{-24}$

b.) $\int_1^1 3f(x)\,dx = \boxed{0}$

c.) $\int_4^6 3f(x)\,dx = 3\int_4^6 f(x)dx = 3\left[\int_1^6 f(x)dx - \int_1^4 f(x)dx\right] = 3(5-8) = 3(-3) = \boxed{-9}$

d.) $\int_6^4 12f(x)\,dx = -\int_4^6 12f(x)dx = -4\int_0^6 3f(x)dx = -4(-9) = \boxed{36}$

5.] Find the average value of $f(x) = x^2$ over the interval $[0, 8]$.

Recall from the previous worksheet:

$\int_0^8 x^2 dx = 170.\overline{6}$

Average value of f:

$\bar{f} = \frac{1}{8-0}\int_0^8 x^2 dx = \frac{1}{8}(170.\overline{6}) = \boxed{21.\overline{3}}$