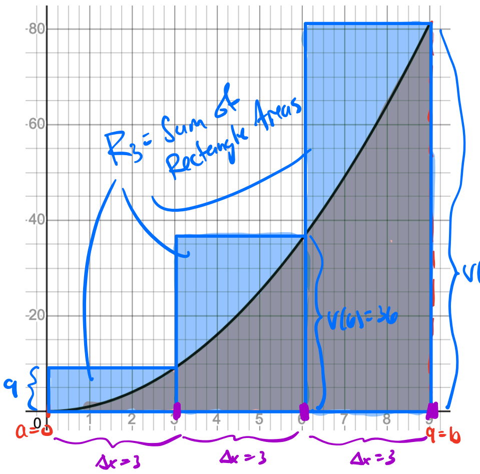
§5.1 (PART 1): AREA, DISTANCE, AND RIEMANN SUMS

1.] Approximate the total distance traveled over 9 hours if the velocity function is given by $v(t) = t^2$.

a.) Use $n = 3$ rectangles to approximate the area with right-hand endpoints.

Bad Approximation



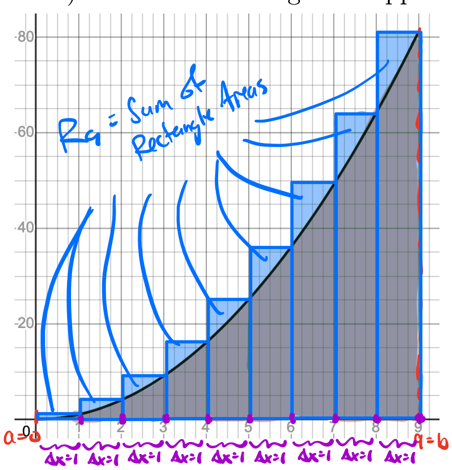
Estimate area under $v(t) = t^2$ on $[0, 9]$ using $n = 3$.

- Interval: $a = 0$, $b = 9$
- Width of Rectangles: $\Delta x = \frac{b-a}{n} = \frac{9-0}{3} = \frac{9}{3} = 3$
- Heights of Rectangles: $v(3) = 3^2 = 9$; $v(6) = 6^2 = 36$; $v(9) = 9^2 = 81$
- Riemann Sum: $R_3 = (9)(3) + (36)(3) + (81)(3)$

$\Rightarrow \boxed{R_3 = 378}$ Total Distance $\approx$ 378 miles

b.) Use $n = 9$ rectangles to approximate the area with right-hand endpoints.

Better Approximation (Less Error)



- Interval: $a = 0$, $b = 9$, $n = 9$
- width: $\Delta x = \frac{b-a}{n} = \frac{9-0}{9} = \frac{9}{9} = 1$
- Heights: $v(1) = 1^2 = 1$, $v(2) = 2^2 = 4$, $v(3) = 3^2 = 9$, $v(4) = 4^2 = 16$, $v(5) = 5^2 = 25$, $v(6) = 6^2 = 36$, $v(7) = 7^2 = 49$, $v(8) = 8^2 = 64$, $v(9) = 9^2 = 81$
- Riemann Sum: $R_9 = (1)(1) + (4)(1) + (9)(1) + (16)(1) + (25)(1) + (36)(1) + \cdots$
$(49)(1) + (64)(1) + (81)(1)$

$\Rightarrow \boxed{R_9 = 285}$ Total Distance $\approx$ 285 miles

c.) Use $n = 18$ rectangles to approximate the area with right-hand endpoints.

- Interval: $a = 0$, $b = 9$, $n = 18$
- width: $\Delta x = \frac{b-a}{n} = \frac{9-0}{18} = \frac{9}{18} = \frac{1}{2}$
- Heights: $v(\frac{1}{2}) = (\frac{1}{2})^2 = \frac{1}{4}$, $v(1) = 1^2 = 1$, $v(\frac{3}{2}) = (\frac{3}{2})^2 = \frac{9}{4}$, $v(2) = 2^2 = 4$, $v(\frac{5}{2}) = (\frac{5}{2})^2 = \frac{25}{4}$, $v(3) = 3^2 = 9$, $v(\frac{7}{2}) = (\frac{7}{2})^2 = \frac{49}{4}$, $v(4) = 4^2 = 16$, $v(\frac{9}{2}) = (\frac{9}{2})^2 = \frac{81}{4}$, $v(5) = 5^2 = 25$, $v(\frac{11}{2}) = (\frac{11}{2})^2 = \frac{121}{4}$, $v(6) = 6^2 = 36$, $v(\frac{13}{2}) = (\frac{13}{2})^2 = \frac{169}{4}$, $v(7) = 7^2 = 49$, $v(\frac{15}{2}) = (\frac{15}{2})^2 = \frac{225}{4}$, $v(8) = 8^2 = 64$, $v(\frac{17}{2}) = (\frac{17}{2})^2 = \frac{289}{4}$, $v(9) = 9^2 = 81$

Best Approximation

- Riemann Sum: $R_{18} = (\frac{1}{4})(\frac{1}{2}) + (1)(\frac{1}{2}) + (\frac{9}{4})(\frac{1}{2}) + (4)(\frac{1}{2}) + (\frac{25}{4})(\frac{1}{2}) + (9)(\frac{1}{2}) + (\frac{49}{4})(\frac{1}{2}) + (16)(\frac{1}{2}) + (\frac{81}{4})(\frac{1}{2}) + \cdots$
$(25)(\frac{1}{2}) + (\frac{121}{4})(\frac{1}{2}) + (36)(\frac{1}{2}) + (\frac{169}{4})(\frac{1}{2}) + (49)(\frac{1}{2}) + (\frac{225}{4})(\frac{1}{2}) + (64)(\frac{1}{2}) + (\frac{289}{4})(\frac{1}{2}) + (81)(\frac{1}{2})$

$\Rightarrow \boxed{R_{18} = 263.625}$ Total Distance $\approx$ 263.625 miles

2.] Calculate the following values using summation notation:

a.) $\sum_{k=1}^{6} k = 1+2+3+4+5+6 = \boxed{21}$

$7 \quad 7 \quad 7$

$= 7 \times 3$

$= \boxed{21}$

b.) $\sum_{k=1}^{n} 5 = \underbrace{\overset{k=1}{5} + \overset{k=2}{5} + \overset{k=3}{5} + \cdots + \overset{k=n}{5}}_{n \text{ fives}}$

$= \boxed{5n}$

c.) $\sum_{k=1}^{4} (2k+1)$

$= (2(1)+1) + (2(2)+1) + (2(3)+1) + (2(4)+1)$

$= 3 + 5 + 7 + 9$

$= \boxed{24}$

$= 2(1+2+3+4) + (1+1+1+1)$

$= 2\sum_{k=1}^{4} k + \sum_{k=1}^{4} 1$

d.) $\sum_{k=1}^{2} (k^2+k) = (1^2+1) + (2^2+2)$

$= \sum_{k=1}^{2} k^2 + \sum_{k=1}^{2} k$

$= (1^2+2^2) + (1+2)$

$\rightarrow 2+6 = \boxed{8}$

3.] Calculate the following values using rules and identities associated with summation notation:

a.) $\sum_{k=1}^{6} k = \frac{(6)(6+1)}{2}$

$= \frac{\cancel{6}^{3} \cdot 7}{\cancel{2}}$

$= 3 \cdot 7$

$= \boxed{21}$

b.) $\sum_{k=1}^{5} (3k-1)$

$= 3\sum_{k=1}^{5} k - \sum_{k=1}^{5} 1$

$= 3\left(\frac{(5)(6)}{2}\right) - 5$

$= 3(15) - 5$

$= 45 - 5$

$= \boxed{40}$

c.) $\sum_{k=1}^{7} (k^3+2k)$

$= \sum_{k=1}^{7} k^3 + 2\sum_{k=1}^{7} k$

$= \left(\frac{7(8)}{2}\right)^2 + 2\left(\frac{7(8)}{2}\right)$

$= 28^2 + 2(28)$

$= 28(28+2)$

$= 28(30)$

$= \boxed{840}$

b.) $\sum_{k=1}^{400} k^2 = \frac{(400)(400+1)(2(800)+1)}{6}$

$= \frac{(400)(401)(801)}{6}$

$= \boxed{21,413,400}$