§4.7: Antiderivatives

1.] Find the antiderivative of the following functions:

a.) $f(x) = 1$ $\longrightarrow$ $F(x) = x + C$

b.) $f(x) = 100x^{99}$ $\longrightarrow$ $F(x) = x^{100} + C$

c.) $h(x) = \sec(x)\tan(x)$ $\longrightarrow$ $H(x) = \sec(x) + C$

d.) $g(x) = \dfrac{1}{x}$ $\longrightarrow$ $G(x) = \ln|x| + C$

e.) $g(x) = -\csc^2(x)$ $\longrightarrow$ $G(x) = \cot(x) + C$

f.) $g(x) = -\dfrac{1}{\sqrt{1-x^2}}$ $\longrightarrow$ $G(x) = \arccos(x) + C$

2.] Find the general antiderivative of $f(x) = e^x - 2x$. Then find the particular antiderivative that satisfies $F(0) = -2$.

$f(x) = e^x - 2x$

Anti Diff

$\boxed{F(x) = e^x - x^2 + C}$

general antiderivative

$F(0) = e^0 - 0^2 + C$

$-2 = 1 - 0 + C$

$-3 = C$

$\boxed{F(x) = e^x - x^2 - 3}$

particular/unique antiderivative

3.] Find the general antiderivative of each of the following functions:

a.) $f(x) = 4x^3 + 1 + \cos(x)$

$F(x) = x^4 + x + \sin(x) + C$

b.) $g(x) = x^2 + 3x + \sin(x)$

$G(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - \cos(x) + C$

c.) $h(x) = 2^x$

$H(x) = \frac{1}{\ln(2)} 2^x + C$

4.] Find the general antiderivative of each of the following functions:

a.) $f(x) = 3x^5 + 2 - 5x^{-2/3}$

$F(x) = \frac{3}{6}x^6 + 2x - \frac{5}{1/3}x^{1/3} + C$

$F(x) = \frac{1}{2}x^6 + 2x - 15x^{1/3} + C$

b.) $g(x) = (x^2 + 1)(2x - 5)$

$g(x) = 2x^3 - 5x^2 + 2x - 5$

$G(x) = \frac{1}{2}x^4 - \frac{5}{3}x^3 + x^2 - 5x + C$

c.) $h(x) = \dfrac{4x^{19} - 5x^{-8}}{x^2}$, where $x \neq 0$.

$h(x) = \frac{4x^{19}}{x^2} - \frac{5x^{-8}}{x^2}$

$h(x) = 4x^{17} - 5x^{-10}$

$H(x) = \frac{4}{18}x^{18} - \frac{5}{-9}x^{-9} + C$

$\boxed{H(x) = \frac{2}{9}x^{18} + \frac{5}{9}x^{-9} + C}$

5.] Solve the initial value problem given by

$$f'(x) = 7x\left(x^6 - \frac{1}{7}\right) \qquad f(1) = 2.$$

$f'(x) = 7x^7 - x$

$f(x) = \frac{7}{8}x^8 - \frac{1}{2}x^2 + C$

$f(1) = \frac{7}{8}(1)^8 - \frac{1}{2}(1)^2 + C$

$2 = \frac{7}{8} - \frac{1}{2} + C$

$C = 2 - \frac{7}{8} + \frac{1}{2}$

$C = \frac{16}{8} - \frac{7}{8} + \frac{4}{8}$

$C = \frac{13}{8}$

$\boxed{f(x) = \frac{7}{8}x^8 - \frac{1}{2}x^2 + \frac{13}{8}}$

6.] Solve the initial value problem given by

$$f''(x) = x + \frac{1}{\sqrt{x}} \qquad f'(4) = 6, \quad f(4) = 0.$$

Anti Diff

$f''(x) = x + x^{-1/2}$

$f'(x) = \frac{1}{2}x^2 + \frac{1}{1/2}x^{1/2} + C$

$f'(x) = \frac{1}{2}x^2 + 2x^{1/2} + C$

$f'(4) = \frac{1}{2}(4)^2 + 2(4)^{1/2} + C$

$6 = 8 + 4 + C$

$-6 = C$

$f'(x) = \frac{1}{2}x^2 + 2x^{1/2} - 6$

$f(x) = \frac{1}{6}x^3 + \frac{2}{3/2}x^{3/2} - 6x + C$

$f(x) = \frac{1}{6}x^3 + \frac{4}{3}x^{3/2} - 6x + C$

$f(4) = \frac{1}{6}(4)^3 + \frac{4}{3}(4)^{3/2} - 6(4) + C$

$0 = \frac{64}{6} + \frac{32}{3} - 24 + C$

$C = \frac{8}{3} \rightarrow \boxed{f(x) = \frac{1}{6}x^3 + \frac{4}{3}x^{3/2} - 6x + \frac{8}{3}}$