§4.3 (Part 1): First Derivative Test

1.] Determine the intervals of monotonicity on for the function $f(x) = -4x^3 - 3x^2 + 18x + 10$.

Critical Pts: $f'(x) = -12x^2 - 6x + 18 = 0$

$-6(2x^2 + x - 3) = 0$

$-6(2x+3)(x-1) = 0$

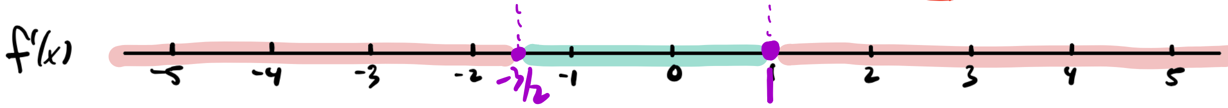
$2x+3=0$ $\quad$ $x-1=0$

$x=-3/2$ $\quad$ $x=1$

$\boxed{c_1 = -3/2 \qquad c_2 = 1}$

$f'(-2) = (-)(-)(-)$ $\qquad$ $f'(0) = 18 > 0$ $\qquad$ $f'(2) = (-)(+)(+)$

$f' < 0$ $\qquad$ $f' > 0$ $\qquad$ $f' < 0$



f is decreasing on $(-\infty, -3/2) \cup (1, \infty)$

f is increasing on $(-3/2, 1)$

2.] Determine the intervals of monotonicity on for the function $f(x) = x^2 e^{-\frac{x}{2}}$.

Critical Pts: $f'(x) = 2xe^{-\frac{x}{2}} + x^2 e^{-\frac{x}{2}} \cdot (-\frac{1}{2})$

$= e^{-x/2}(2x - \frac{x^2}{2})$

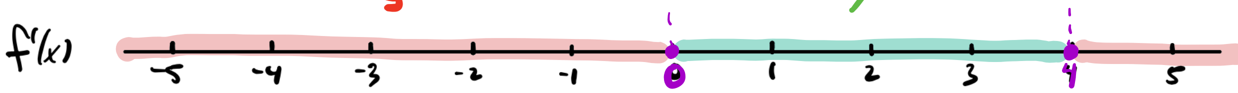
$= \frac{1}{2} e^{-\frac{x}{2}} x (4-x) = 0$

$x=0$ $\quad$ $4-x=0$

$\boxed{c_1 = 0 \qquad c_2 = 4}$

$f'(-1) = (+)(-)(+)$ $\qquad$ $f'(1) = (+)(+)(+)$ $\qquad$ $f'(5) = (+)(+)(-)$

$f' < 0$ $\qquad$ $f' > 0$



f is decreasing on $(-\infty, 0) \cup (4, \infty)$

f is increasing on $(0, 4)$

3.] Determine the intervals of monotonicity on for the function $f(x) = x^{2/3}(5-x)$. Then use the First Derivative Test to determine the local extrema.

Critical Pts: $f'(x) = \frac{2}{3}x^{-1/3}(5-x) + x^{2/3}(-1)$

$$= \frac{2(5-x)}{3x^{1/3}} - x^{2/3}$$

$$= \frac{10-2x - x^{2/3}(3x^{1/3})}{3x^{1/3}}$$

$$= \frac{10-2x-3x}{3x^{1/3}} = \frac{10-5x}{3x^{1/3}} = \frac{5(2-x)}{3x^{1/3}}$$

$f'(x) = \text{DNE}$: $3x^{1/3} = 0 \rightarrow x = 0$

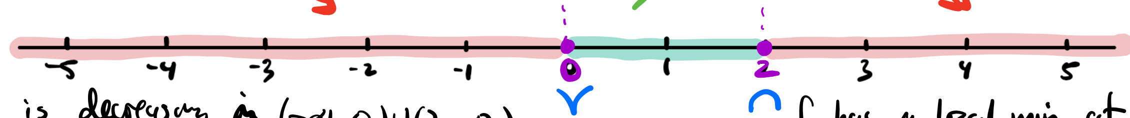
$f'(x) = 0$: $5(2-x) = 0 \rightarrow x = 2$

$\boxed{\text{Critical Pts: } c_1 = 0,\ c_2 = 2}$

$f'(-1) = \frac{5(2-(-1))}{3(-1)^{1/3}} = \frac{15}{-3} = -5 < 0$ $f'(1) = \frac{(+)}{(+)}$ $f'(3) = \frac{(-)}{(+)}$

$f' < 0$ $f' > 0$ $f' < 0$

$f'(x)$



f is decreasing on $(-\infty, 0) \cup (2, \infty)$

f is increasing on $(0, 2)$

f has a local min at $x = 0$ (cusp)

f has a local max at $x = 2$

4.] Determine the intervals of monotonicity of the function $f(x) = 2x^5 - 5x^4 - 10x^3 + 4$. Use the First Derivative Test to locate the local extrema, and identify the absolute maximum and minimum values of the function on $[-2, 4]$.

Critical Pts: $f'(x) = 10x^4 - 20x^3 - 30x^2$

$f'(x) = 10x^2(x^2 - 2x - 3)$

$\boxed{f'(x) = 10x^2(x-3)(x+1)}$

$10x^2(x-3)(x+1) = 0$

$10x^2 = 0$ $x - 3 = 0$ $x + 1 = 0$

$x = 0$ $x = 3$ $x = -1$

$\Rightarrow \boxed{c_1 = -1,\ c_2 = 0,\ c_3 = 3}$

Closed Interval Method:

$f(-2) = -60$

$f(-1) = 7$

$f(3) = -185$ *

$f(4) = 132$ *

Abs Min at $x = 3$

Abs Max at $x = 4$

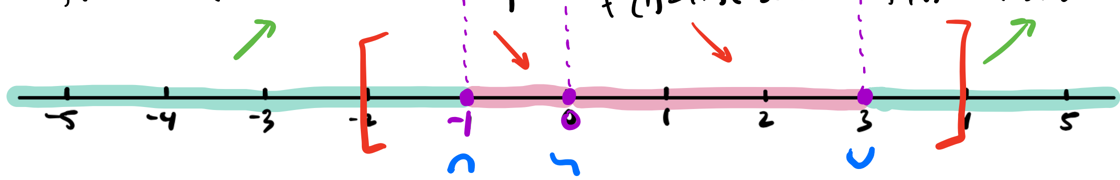
First Derivative Test:

local max at $x = -1$

local min at $x = 3$

$f'(-2) = (+)(-)(-) > 0$ $f'(-\frac{1}{2}) = (+)(-)(+) < 0$ $f'(1) = (+)(-)(+) < 0$ $f'(4) = (+)(+)(+)$

$f'(x)$



f is decreasing on $(-1, 3)$

f is increasing on $(-\infty, -1) \cup (3, \infty)$