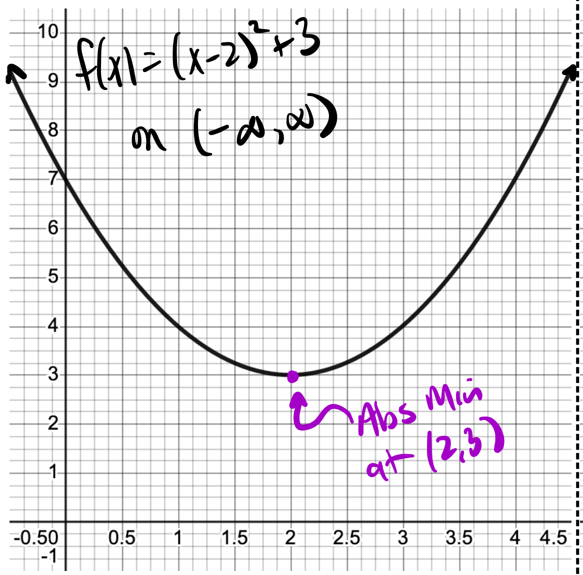
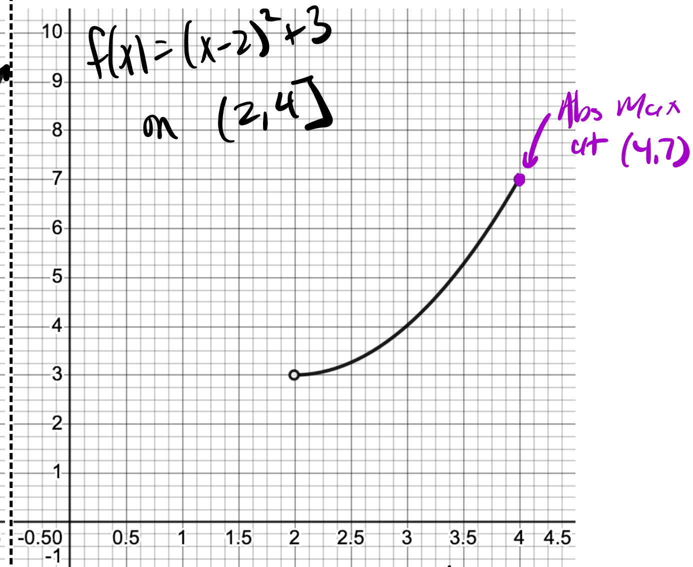
§4.1: Extreme Values of Functions

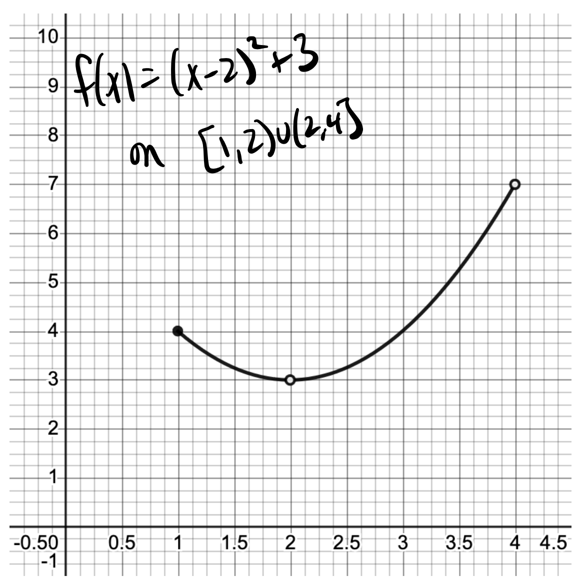
1.] In the graphs below, identify the points (if any) on the interval given if the function has an absolute minimum or an absolute maximum.



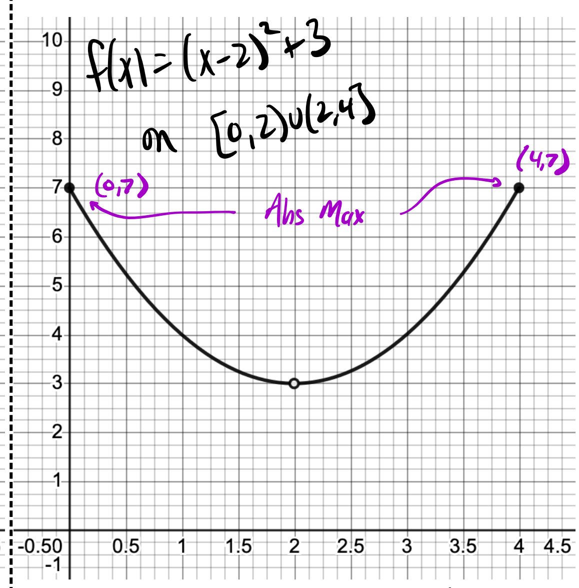
- No Absolute Maximum.
- Absolute Minimum value of $y=3$, which occurs at $x=2$.



- Absolute Maximum value of $y=7$, which occurs at $x=4$.
- No Absolute Minimum.



- No Absolute Maximum.
- No Absolute Minimum.



- Absolute Maximum value of $y=7$, which occurs at $x=0$ and $x=4$.
- No Absolute Minimum.

2.] Find the critical points of the following functions on the domain or on the given interval.

a.) $f(x) = \dfrac{x^3}{3} - 9x$ on $[-7, 7]$

$f'(x) = \dfrac{3x^2}{3} - 9$

$f'(x) = x^2 - 9$

$\underline{f'(x) = DNE}$

N/A, always exists

$\underline{f'(x) = 0}$:

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3, 3$

Critical Pts:

$\boxed{c_1 = -3,\ c_2 = 3}$

b.) $f(x) = x^2\sqrt{x+5}$ Domain: $[-5, \infty)$

$f'(x) = 2x\sqrt{x+5} + x^2 \dfrac{1}{2\sqrt{x+5}}$

$f'(x) = \dfrac{2x\sqrt{x+5}\,(2\sqrt{x+5}) + x^2}{2\sqrt{x+5}}$

$f'(x) = \dfrac{4x(x+5) + x^2}{2\sqrt{x+5}}$

$f'(x) = \dfrac{5x^2 + 20x}{2\sqrt{x+5}}$

$\underline{f'(x) = 0}$:

$5x^2 + 20x = 0$

$5x(x+4) = 0$

$x = 0 \quad x = -4$

Critical Pts:

$\boxed{c_1 = -4,\ c_2 = 0}$

$\underline{f'(x) = DNE}$

$2\sqrt{x+5} = 0$

$\Rightarrow x = -5$ Not interior

3.] Find the critical points of the function $f(x) = 3x^5 - 25x^3 + 60x$ on the interval $[-2, 3]$. Determine the absolute extreme values of f on the given interval.

1.) Critical Pts: $f'(x) = 15x^4 - 75x^2 + 60$

$15x^4 - 75x^2 + 60 = 0$

$\Rightarrow 15(x^4 - 5x^2 + 4) = 0$

$\Rightarrow x^4 - 5x^2 + 4 = 0$

$\Rightarrow (x^2 - 1)(x^2 - 4) = 0$

$\Rightarrow (x-1)(x+1)(x-2)(x+2) = 0$

$x = -2, -1, 1, 2$

$c_1 = -1,\ c_2 = 1,\ c_3 = 2$

2.) $f(a) = f(-2) = 3(-2)^5 - 25(-2)^3 + 60(-2) = -16$

min

$f(c_1) = f(-1) = 3(-1)^5 - 25(-1)^3 + 60(-1) = -38$

$f(c_2) = f(1) = 3(1)^5 - 25(1)^3 + 60(1) = 38$

$f(c_3) = f(2) = 3(2)^5 - 25(2)^3 + 60(2) = 16$

$f(b) = f(3) = 3(-2)^5 - 25(-2)^3 + 60(-2) = 234$

max

3.) $\boxed{\text{Abs min at } (-1, -38) \quad \text{Abs max at } (3, 234)}$