§4.14: Unrestricted-in-Sign Variables

1.] Use the simplex algorithm to solve the following LP:

$$\text{Maximize:} \quad z = 2x_1 + x_2$$

$$\text{Subject to:} \quad 3x_1 + x_2 \le 6$$
$$x_1 + x_2 \le 4$$
$$x_1 \ge 0, x_2 \text{ unrestricted}$$

Define $x_2 = x_2' - x_2''$, then

Maximize $z = 2x_1 + x_2' - x_2''$

Subject to $3x_1 + x_2' - x_2'' \le 6 \Rightarrow$

$x_1 + x_2' - x_2'' \le 4$

$x_1, x_2', x_2'' \ge 0$

Maximize $z = 2x_1 + x_2' - x_2''$

Subject to $3x_1 + x_2' - x_2'' + s_1 = 6$

$x_1 + x_2' - x_2'' + s_2 = 4$

$x_1, x_2', x_2'' \ge 0$

Row	Basic	z	x_1	x_2'	x_2''	s_1	s_2	RHS
0	z	1	-2	-1	1	0	0	0
1	s_1	0	3	1	-1	1	0	6
2	s_2	0	1	1	-1	0	1	4

$6/3 = 2$ ✓

$4/1 = 4$

Row	Basic	z	x_1	x_2'	x_2''	s_1	s_2	RHS
0'	z	1	0	$-1/3$	$1/3$	$2/3$	0	4
1'	x_1	0	1	$1/3$	$-1/3$	$1/3$	0	2
2'	s_2	0	0	$2/3$	$-2/3$	$-1/3$	1	2

$\frac{2}{1/3} = 6$

$\frac{2}{2/3} = 3$ ✓

Row	Basic	z	x_1	x_2'	x_2''	s_1	s_2	RHS
0"	z	1	0	0	0	$1/2$	$1/2$	5
1"	x_1	0	1	0	0	$1/2$	$-1/2$	1
2"	x_2'	0	0	1	-1	$-1/2$	$3/2$	3

optimal!

Solution: $x_1 = 1$, $x_2' = 3$, $x_2'' = 0$, Max $z = 5$.

$x_2 = 3$

2.] PantCo manufactures pants. During each of the next six months they can sell up to the numbers of pants given in the table below. Demand that is not met during a month is lost. Thus, for example, PantCo can sell up to 500 pants during month 1. A pair of pants sells for \$40, requires 2 hours of labor, and uses \$10 of raw material. At the beginning of month 1, PantCo has 4 workers. A worker can work at making pants up to 200 hours per month, and is paid \$2000 per month (irrespective of how many hours they work). At the beginning of each month, workers can be hired or fired. It costs \$1500 to hire and \$1000 to fire a worker. A holding cost of \$5 per pair of pants is assess against each month's inventory. Determine how PantCo can maximize its profit for the next six months. Ignore the fact that during each month the number of hired and fired workers must be an integer.

Month	Max Demand
1	500
2	600
3	300
4	400
5	300
6	800

Decision Variables:

P_t = pairs of pants produced in month t $(P_t \geq 0)$

S_t = pairs of pants sold in month t $(S_t \geq 0)$

i_t = pairs of pants in inventory at end of month t. $(i_t \geq 0)$

w_t = workers at the beginning of month t $(w_t \geq 0)$

Δw_t = change in workers at beginning of month t. $(\Delta w_t = \text{urs})$

↳ $\Delta w_t = h_t - f_t$ = (# of workers hired) − (# of workers fired) $(h_t \geq 0, f_t \geq 0)$

Constraints:

(workers)

$w_1 = 4 + \Delta w_1$ $\quad w_1 = 4 + h_1 - f_1$

$w_2 = w_1 + \Delta w_2$ $\quad w_2 = w_1 + h_2 - f_2$

$w_3 = w_2 + \Delta w_3$ $\Rightarrow$ $w_3 = w_2 + h_3 - f_3$

$w_4 = w_3 + \Delta w_4$ $\quad w_4 = w_3 + h_4 - f_4$

$w_5 = w_4 + \Delta w_5$ $\quad w_5 = w_4 + h_5 - f_5$

$w_6 = w_5 + \Delta w_6$ $\quad w_6 = w_5 + h_6 - f_6$

(Demand)

$S_1 \leq 500$

$S_2 \leq 600$

$S_3 \leq 300$

$S_4 \leq 400$

$S_5 \leq 300$

$S_6 \leq 800$

(Labor)

$2P_1 \leq 200 w_1$

$2P_2 \leq 200 w_2$

$2P_3 \leq 200 w_3$

$2P_4 \leq 200 w_4$

$2P_5 \leq 200 w_5$

$2P_6 \leq 200 w_6$

(Inventory)

$i_1 = P_1 - S_1$

$i_2 = P_2 - S_2 + i_1$

$i_3 = P_3 - S_3 + i_2$

$i_4 = P_4 - S_4 + i_3$

$i_5 = P_5 - S_5 + i_4$

$i_6 = P_6 - S_6 + i_5$

Objective function: Maximize Profit

$$\text{Maximize } Z = 40 \sum_{t=1}^{6} S_t - \cdots$$

$$2000 \sum_{t=1}^{6} w_t - \cdots$$

$$1500 \sum_{t=1}^{6} h_t - \cdots$$

$$1000 \sum_{t=1}^{6} f_t - \cdots$$

$$5 \sum_{t=1}^{6} i_t - \cdots$$

$$10 \sum_{t=1}^{6} P_t$$