§3.4: The Chain Rule

1.] Differentiate the following functions:

a.) $f(x) = (\underbrace{2x-3}_{g(x)})^4$

$f'(x) = 4(2x-3)^3 \cdot (2x-3)'$

$f'(x) = 4(2x-3)^3 \cdot 2$

$\boxed{f'(x) = 8(2x-3)^3}$

b.) $f(x) = \sqrt{\underbrace{\cos(x)}_{g(x)}}$

$f'(x) = \dfrac{1}{2\sqrt{\cos(x)}} \cdot (\cos(x))'$

$\boxed{f'(x) = \dfrac{-\sin(x)}{2\sqrt{\cos(x)}}}$

c.) $f(x) = \cos(\underbrace{\sqrt{x}}_{g(x)})$

$f'(x) = -\sin(\sqrt{x}) \cdot (\sqrt{x})'$

$f'(x) = -\sin(\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}}$

$\boxed{f'(x) = -\dfrac{\sin(\sqrt{x})}{2\sqrt{x}}}$

d.) $f(x) = e^{\overbrace{3x}^{g(x)}}$

$f'(x) = e^{3x} \cdot (3x)'$

$f'(x) = e^{3x} \cdot 3$

$\boxed{f'(x) = 3e^{3x}}$

2.] Differentiate the following functions:

a.) $h(x) = \sqrt{\underbrace{\tan(\underbrace{5x^2}_{h(x)})}_{g(x)}}$

$h'(x) = \dfrac{1}{2\sqrt{\tan(5x^2)}} \cdot (\tan(5x^2))'$

$h'(x) = \dfrac{1}{2\sqrt{\tan(5x^2)}} \cdot \sec(5x^2) \cdot (5x^2)'$

$h'(x) = \dfrac{1}{2\sqrt{\tan(5x^2)}} \cdot \sec^2(5x^2) \cdot 10x$

$h'(x) = \dfrac{10x \sec^2(5x^2)}{2\sqrt{\tan(5x^2)}}$

$\boxed{h'(x) = \dfrac{5x \sec^2(5x^2)}{\sqrt{\tan(5x^2)}}}$

b.) $k(x) = \underbrace{\csc(4x^2)}_{f}\underbrace{2^{\sec(x)}}_{g}$

$k'(x) = (\csc(4x^2))'(2^{\sec(x)}) + (2^{\sec(x)})'(\csc(4x^2))$

$k'(x) = -\csc(4x^2)\cot(4x^2) \cdot (4x^2)' \cdot 2^{\sec(x)} + \dots$
$\ln(2) \cdot 2^{\sec(x)} \cdot (\sec(x))' \csc(4x^2)$

$k'(x) = -\csc(4x^2)\cot(4x^2) \cdot 8x \cdot 2^{\sec(x)} + \dots$
$\ln(2) \cdot 2^{\sec(x)} \cdot \sec(x)\tan(x) \csc(4x^2)$

$\boxed{k'(x) = 2^{\sec(x)}\csc(4x^2)\left(-8x\cot(4x^2) + \ln(2)\sec(x)\tan(x)\right)}$

3.] Differentiate the following function: $f(x) = \left(\cos^4(7x^3) + 4\sec(x)\right)^6$.

$$f'(x) = 6\left(\cos^4(7x^3)+4\sec(x)\right)^5 \cdot \left(\cos^4(7x^3)+4\sec(x)\right)'$$

$$f'(x) = 6\left(\cos^4(7x^3)+4\sec(x)\right)^5 \cdot \left(4\cos^3(7x^3)\cdot(\cos(7x^3))' + 4\sec(x)\tan(x)\right)$$

$$f'(x) = 6\left(\cos^4(7x^3)+4\sec(x)\right)^5 \cdot \left(4\cos^3(7x^3)\cdot(-\sin(7x^3))\cdot(7x^3)' + 4\sec(x)\tan(x)\right)$$

$$\boxed{f'(x) = 6\left(\cos^4(7x^3)+4\sec(x)\right)^5 \cdot \left(4\cos^3(7x^3)\cdot(-\sin(7x^3))\cdot 21x^2 + 4\sec(x)\tan(x)\right)}$$

4.] Find the equation of the tangent line to the graph of $f(x) = \left(x^2 + 3x + 4\right)^{2/3}$ at the point $(1, f(1))$.

Equation of Tangent Line:

$$y - f(c) = f'(c)(x - c)$$

• $c = \boxed{1}$

• $f(c) = f(1)$

$= \left(1^2 + 3(1) + 4\right)^{2/3}$

$= (8)^{2/3}$

$= (2)^2$

$= \boxed{4}$

• $f'(x) = \frac{2}{3}\left(x^2 + 3x + 4\right)^{-1/3} \cdot (2x+3)$

$= \dfrac{2(2x+3)}{3\sqrt[3]{x^2+3x+4}}$

• $f'(1) = \dfrac{2(2(1)+3)}{3\sqrt[3]{1^2+3(1)+4}}$

$= \dfrac{2(5)}{3\sqrt[3]{8}}$

$= \dfrac{10}{3\cdot 2}$

$= \boxed{\dfrac{5}{3}}$

$\Rightarrow \quad y - 4 = \frac{5}{3}(x - 1)$

$\Rightarrow \quad y - 4 = \frac{5}{3}x - \frac{5}{3}$

$\Rightarrow \quad y = \frac{5}{3}x - \frac{5}{3} + \frac{12}{3}$

$\Rightarrow \quad \boxed{y = \frac{5}{3}x + \frac{7}{3}}$