§3.2 (part 2): Derivatives of Products and Quotients

1.] Differentiate the following functions:

a.) $f(x) = (2x+1)(x^2-1)$

$f'(x) = (2x+1)'(x^2-1) + (2x+1)(x^2-1)'$

$f'(x) = (2)(x^2-1) + (2x+1)(2x)$

$\Rightarrow f'(x) = 2x^2-2+4x^2+2x$

$\Rightarrow \boxed{f'(x) = 6x^2+2x-2}$

b.) $g(x) = x^2e^x$

$g'(x) = (x^2)'(e^x) + (x^2)(e^x)'$

$\Rightarrow g'(x) = 2xe^x + x^2e^x$

$\Rightarrow \boxed{g'(x) = xe^x(2+x)}$

c.) $h(x) = 2^x(x^3-\sqrt{x})$

$h'(x) = (2^x)'(x^3-\sqrt{x}) + (2^x)(x^3-\sqrt{x})'$

$\Rightarrow h'(x) = \ln(2)\cdot 2^x(x^3-\sqrt{x}) + 2^x\left(3x^2-\frac{1}{2\sqrt{x}}\right)$

$\Rightarrow \boxed{h'(x) = 2^x\left(\ln(2)x^3-\ln(2)\sqrt{x}+3x^2-\frac{1}{2\sqrt{x}}\right)}$

d.) $k(x) = 3^2\cdot 4^3$

$k(x) = 9\cdot 64$

$k(x) = 576$

$\boxed{k'(x) = 0}$

2.] Differentiate the following functions:

a.) $f(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{(x-1)(x^2)' - (x^2)(x-1)'}{(x-1)^2}$

$\Rightarrow f'(x) = \dfrac{(x-1)(2x) - (x^2)(1)}{(x-1)^2}$

$\Rightarrow \boxed{f'(x) = \dfrac{x^2-2x}{(x-1)^2}}$

b.) $g(x) = \dfrac{x^3-x}{x}$

$g'(x) = \dfrac{(x)(x^3-x)' - (x^3-x)(x)'}{x^2}$

$\Rightarrow g'(x) = \dfrac{x(3x^2-1) - (x^3-x)}{x^2}$

$\Rightarrow g'(x) = \dfrac{3x^3-x-x^3+x}{x^2}$

$\Rightarrow \boxed{g'(x) = \dfrac{2x^3}{x^2}}$

c.) $h(x) = \dfrac{x^2+3x+4}{x^2-1}$

$h'(x) = \dfrac{(x^2-1)(x^2+3x+4)' - (x^2+3x+4)(x^2-1)'}{(x^2-1)^2}$

$\Rightarrow h'(x) = \dfrac{(x^2-1)(2x+3) - (x^2+3x+4)(2x)}{(x^2-1)^2}$

$\Rightarrow h'(x) = \dfrac{2x^3+3x^2-2x-3-2x^3-6x^2-8x}{(x^2-1)^2}$

$\Rightarrow \boxed{h'(x) = \dfrac{-3x^2-10x-3}{(x^2-1)^2}}$

d.) $k(x) = \dfrac{2^x}{2^x+1}$

$k'(x) = \dfrac{(2^x+1)(2^x)' - (2^x)(2^x+1)'}{(2^x+1)^2}$

$\Rightarrow k'(x) = \dfrac{(2^x+1)(\ln(2)2^x) - 2^x(\ln(2)\cdot 2^x)}{(2^x+1)^2}$

$\Rightarrow k'(x) = \dfrac{\cancel{\ln(2)(2^x)^2} + \ln(2)2^x - \cancel{\ln(2)(2^x)^2}}{(2^x+1)^2}$

$\Rightarrow \boxed{k'(x) = \dfrac{\ln(2)\cdot 2^x}{(2^x+1)^2}}$

3.] Differentiate the following functions:

a.) $f(x) = \dfrac{x^2}{3^x}$

$$f'(x) = \frac{(3^x)(x^2)' - (x^2)(3^x)'}{(3^x)^2}$$

$$\Rightarrow f'(x) = \frac{3^x 2x - x^2 \ln(3)\cdot 3^x}{(3^x)^2}$$

$$\Rightarrow \boxed{f'(x) = \frac{2x - \ln(3)\cdot x^2}{3^x}}$$

b.) $g(x) = \dfrac{4xe^x}{x^2+1}$

$$g'(x) = \frac{(x^2+1)(4xe^x)' - (4xe^x)(x^2+1)'}{(x^2+1)^2}$$

$$\Rightarrow g'(x) = \frac{(x^2+1)(4e^x + 4xe^x) - 4xe^x(2x)}{(x^2+1)^2}$$

$$\Rightarrow g'(x) = \frac{4x^2e^x + 4x^3e^x + 4e^x + 4xe^x - 8x^2e^x}{(x^2+1)^2}$$

$$\Rightarrow \boxed{g'(x) = \frac{4e^x(x^3 - x^2 + x + 1)}{(x^2+1)^2}}$$

4.] Find the equation of the line tangent to the graph of the function $f(x) = \dfrac{8}{x^2+4}$ at the point $(2, f(2))$.

We need to find $f(2)$ and $f'(2)$.

$$f(x) = \frac{8}{x^2+4}$$

$$f(2) = \frac{8}{2^2+4} = \frac{8}{4+4} = \frac{8}{8} = 1$$

$$\boxed{f(2) = 1}$$

$$f'(x) = \frac{(x^2+4)(8)' - (8)(x^2+4)'}{(x^2+4)^2} = \frac{0 - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(2^2+4)^2} = \frac{-32}{8^2} = \frac{-32}{64} = -\frac{1}{2}$$

$$\boxed{f'(2) = -\frac{1}{2}}$$

Eqn of TL at $(2, f(2))$:

$$y - f(2) = f'(2)(x-2)$$

$$\Rightarrow y - 1 = -\frac{1}{2}(x-2)$$

$$\Rightarrow y - 1 = -\frac{1}{2}x + 1$$

$$\Rightarrow \boxed{y = -\frac{1}{2}x + 2}$$