# §3.2 (part 1): Derivatives of Polynomials and Exponentials

1.] Differentiate the following functions:

*a.)* $f(x) = \pi$

$f'(x) = 0$

($\pi$ is a constant)

*b.)* $g(x) = x^7$

$g'(x) = 7x^{7-1}$

$g'(x) = 7x^6$

*c.)* $h(x) = x$

$h'(x) = 1x^{1-1}$

$h'(x) = 1x^0$

$h'(x) = 1$

*d.)* $k(x) = \sqrt{x}$

$k'(x) = \frac{1}{2\sqrt{x}}$

we did this using limit definition on a previous worksheet

But look: $k(x) = x^{1/2}$

↳ Power Rule → $k'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ ✓

2.] Differentiate the following functions, using the correct notation:

*a.)* $f(x) = 6x^5 - x$

$f'(x) = 6(5x^4) - 1x^0$

$\Rightarrow f'(x) = 30x^4 - 1$

*b.)* $y = 6\sqrt{x} - 4x^3 + 9$

$\frac{dy}{dx} = 6\left(\frac{1}{2\sqrt{x}}\right) - 4(3x^2) + 0$

$\Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{x}} - 12x^2$

*c.)* $f(x) = 4\sqrt{x} - \frac{1}{4}x^4 + x + 1$

$f'(x) = 4\left(\frac{1}{2\sqrt{x}}\right) - \frac{1}{4}(4x^3) + 1x^0 + 0$

$\Rightarrow f'(x) = \frac{2}{\sqrt{x}} - x^3 + 1$

*d.)* $y = \sqrt{x}(\sqrt{x} - 1)$

$\Rightarrow y = x - \sqrt{x}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x}}$

*e.)* $f(x) = (x^2 + 1)^2$

$\Rightarrow f(x) = (x^2+1)(x^2+1)$

$\Rightarrow f(x) = x^4 + 2x^2 + 1$

$\Rightarrow f'(x) = 4x^3 + 2(2x^1) + 0$

$\Rightarrow f'(x) = 4x^3 + 4x$

*f.)* $y = (2x + 1)(3x^2 + 2)$

$\Rightarrow y = 6x^3 + 4x + 3x^2 + 2$

$\Rightarrow \frac{dy}{dx} = 6(3x^2) + 4(1x^0) + 3(2x^1) + 0$

$\Rightarrow \frac{dy}{dx} = 18x^2 + 4 + 6x$

3.] Determine the following limits:

a.) $\lim_{h\to 0} \frac{e^h - 1}{h}$ $= 1$

$b = e$

b.) $\lim_{h\to 0} \frac{2^h - 1}{h}$ $= \ln(2)$

$b = 2$

4.] Find the first and second derivatives of the following functions:

*a.)* $f(x) = 4x^2 - x + 1$

$f'(x) = 8x - 1$

$f''(x) = 8$

*b.)* $g(x) = 3e^x + x^{100} - 3x$

$g'(x) = 3e^x + 100x^{99} - 3$

$g''(x) = 3e^x + 9900x^{98}$

*c.)* $h(x) = 4^x + 4^2$

$\Rightarrow h'(x) = \ln(4)\cdot 4^x + 0$

$\Rightarrow h'(x) = \ln(4)\cdot 4^x$

$\Rightarrow h''(x) = \ln(4)\cdot(\ln(4)\cdot 4^x)$

$\Rightarrow h''(x) = \ln^2(4)\cdot 4^x$

*d.)* $k(x) = 3^{2x}$

$\Rightarrow k(x) = (3^2)^x$

$\Rightarrow k(x) = 9^x$

$\Rightarrow k'(x) = \ln(9)\cdot 9^x$

$\Rightarrow k''(x) = \ln(9)\cdot(\ln(9)\cdot 9^x)$

$\Rightarrow k''(x) = \ln^2(9)\cdot 9^x$

5.] Find the equation of the tangent line to the curve $y = -3x^2 + 2$ at the point $(2, f(2))$.

Function: $f(x) = -3x^2 + 2$

Point: $(x_1, y_1) = (2, f(2)) = (2, -10)$

$f(2) = -3(2)^2 + 2 = -3(4) + 2 = -12 + 2 = -10$

Slope: $m = f'(2) = -12$

$f'(x) = -3(2x) = -6x$

$f'(2) = -6(2) = -12$

Eqn of TL: $y - y_1 = m(x - x_1)$

$\Rightarrow y - (-10) = -12(x - 2)$

$\Rightarrow y + 10 = -12x + 24$

$\Rightarrow y = -12x + 14$