§2.5: Double-Angle Formulas and Product-to-Sum Formulas

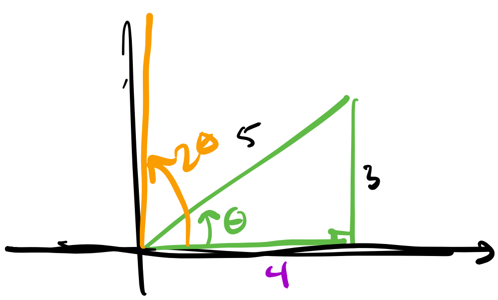
1.] Use a double-angle formula to evaluate $\sin\left(\frac{\pi}{3}\right)$ exactly.

$$\sin(60^\circ) = \sin(2\cdot 30^\circ) = 2\sin(30^\circ)\cos(30^\circ)$$
$$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

$\sin(2x) = 2\sin(x)\cos(x)$

2.] Suppose $\sin(\theta) = \dfrac{3}{5}$ and θ is in the first quadrant. Find $\sin(2\theta)$, $\cos(2\theta)$ and $\tan(2\theta)$.

$\sin(\theta) = \frac{3}{5}$



$\cos(\theta) = \frac{4}{5}$

$\tan(\theta) = \frac{3}{4}$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{\frac{24}{25}}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{3/2}{1-\frac{9}{16}} = \cdots$$

$$\cdots \frac{3/2}{7/16} = \frac{3\cdot \cancel{16}^{\,8}}{7\cdot \cancel{2}} = \boxed{\frac{24}{7}}$$

3.] Find the general solution to the equation: $2\cos(x) + \sin(2x) = 0$

$$2\cos(x) + 2\sin(x)\cos(x) = 0$$
$$2\cos(x)\left(1 + \sin(x)\right) = 0$$

$2\cos(x) = 0$ $\qquad$ $1 + \sin(x) = 0$

$\cos(x) = 0$ $\qquad$ $\sin(x) = -1$

$x = \pi/2,\ 3\pi/2$ $\qquad$ $x = 3\pi/2$

$$\boxed{x = \frac{\pi}{2} + n2\pi, \quad x = \frac{3\pi}{2} + n2\pi}$$

4.] Verify the following identity: $\underbrace{\big(\sin(x)+\cos(x)\big)^2}_{LHS} = \underbrace{1+\sin(2x)}_{RHS}$.

LHS: $(\sin(x)+\cos(x))^2 = (\sin(x)+\cos(x))(\sin(x)+\cos(x))$

$= \sin^2(x) + \sin(x)\cos(x) + \cos(x)\sin(x) + \cos^2(x)$

$= \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x)$

$= \sin^2(x) + \cos^2(x) + 2\sin(x)\cos(x)$

$= 1 + 2\sin(x)\cos(x)$

$= 1 + \sin(2x)$: RHS

$\sin(2x) = 2\sin(x)\cos(x)$

5.] Write the expression $\cos^4(x)$ in terms of first powers of cosine.

$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$\cos^4(x) = \cos^2(x)\cos^2(x)$

$= \left(\frac{1}{2}+\frac{1}{2}\cos(2x)\right)\left(\frac{1}{2}+\frac{1}{2}\cos(2x)\right)$

$= \frac{1}{4} + \frac{1}{4}\cos(2x) + \frac{1}{4}\cos(2x) + \frac{1}{4}\cos^2(2x)$

$= \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)$

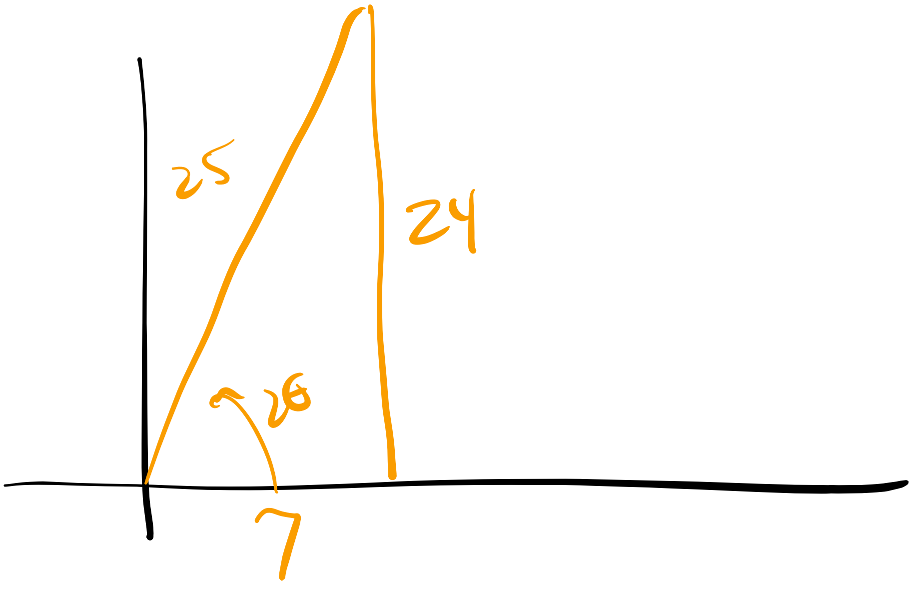
$= \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos(4x)\right)$

$= \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x) = \boxed{\frac{3}{8} + \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)}$

6.] Use a product-to-sum formula to write $\sin(5\theta)\sin(3\theta)$ as a sum or difference.

$\sin(\overset{u}{5\theta})\sin(\overset{v}{3\theta}) = \frac{1}{2}\cos(5\theta-3\theta) - \frac{1}{2}\cos(5\theta+3\theta)$

$= \frac{1}{2}\cos(2\theta) - \frac{1}{2}\cos(8\theta)$


25
24
2θ
7