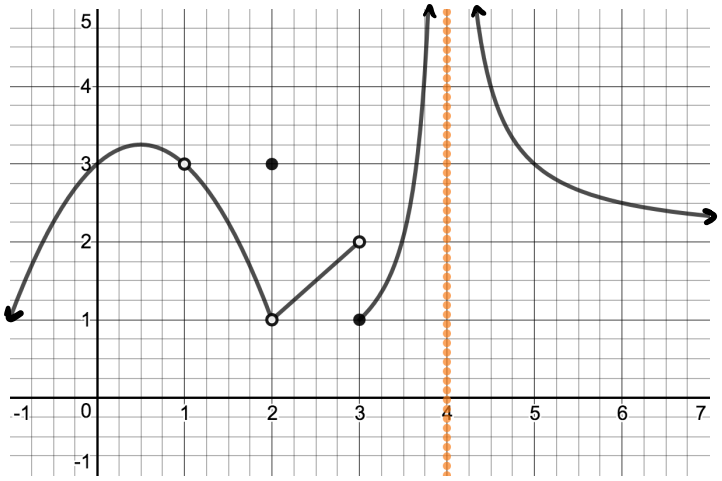
§2.5 (PART 1): CONTINUITY

1.] Consider the graph of a piecewise function $f(x)$ below:



Domain: $x \neq 1, 4$

$(-\infty, 1) \cup (1, 4) \cup (4, \infty)$

Continuous on

$(-\infty, 1) \cup (1, 2) \cup (2, 3) \cup (3, 4) \cup (4, \infty)$

a.) Compute the following:

i. $\lim_{x \to 1^-} f(x) = 3$ *ii.* $\lim_{x \to 1^+} f(x) = 3$ *iii.* $\lim_{x \to 1} f(x) = 3$ *iv.* $f(1) =$ DNE

Not continuous at $x=1$ → Failed Part 1

Removable Discontinuities

b.) Compute the following:

i. $\lim_{x \to 2^-} f(x) = 1$ *ii.* $\lim_{x \to 2^+} f(x) = 1$ *iii.* $\lim_{x \to 2} f(x) = 1$ *iv.* $f(2) = 3$

Not continuous at $x=2$ → Failed Part 3

c.) Compute the following:

i. $\lim_{x \to 3^-} f(x) = 2$ *ii.* $\lim_{x \to 3^+} f(x) = 1$ *iii.* $\lim_{x \to 3} f(x) =$ DNE *iv.* $f(3) = 1$

Not continuous at $x=3$ → Failed Part 2

Jump Discontinuity

d.) Compute the following:

i. $\lim_{x \to 4^-} f(x) = \infty$ *ii.* $\lim_{x \to 4^+} f(x) = \infty$ *iii.* $\lim_{x \to 4} f(x) = \infty$ *iv.* $f(4) =$ DNE

Not continuous at $x=4$ → Failed Parts 1 & 2

Vertical Asymptote (Singularity)

e.) Compute the following:

i. $\lim_{x \to 6^-} f(x) = 2.5$ *ii.* $\lim_{x \to 6^+} f(x) = 2.5$ *iii.* $\lim_{x \to 6} f(x) = 2.5$ *iv.* $f(6) = 2.5$

Continuous at $x=6$!

2.] Determine if the following functions are continuous at c:

a.) $f(x) = \sqrt{x-2}, \qquad c = 1$

1.) $f(1) = \sqrt{1-2} = \sqrt{-1} = i$ ✗

2.) N/A

3.) N/A

Not Continuous at $c=1$.

b.) $f(x) = \dfrac{2x^2+3x+1}{x^2+5x}, \qquad c = 5$

1.) $f(5) = \dfrac{2(5)^2+3(5)+1}{5^2+5(5)} = \dfrac{50+15+1}{25+25} = \dfrac{66}{50} = \dfrac{33}{25}$ ✓

2.) $\lim\limits_{x\to 5} \dfrac{2x^2+3x+1}{x^2+5x} = \dfrac{2(5)^2+3(5)+1}{5^2+5(5)} = \dfrac{50+15+1}{25+25} = \dfrac{33}{25}$ ✓

3.) $\dfrac{33}{25} = \dfrac{33}{25}$ ✓

Continuous at $c=5$!

c.) $f(x) = \begin{cases} \dfrac{x^2+x}{x+1} & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases}, \qquad c = -1$

1.) $f(-1) = 2$ ✓

2.) $\lim\limits_{x\to -1} \dfrac{x^2+x}{x+1} = \lim\limits_{x\to -1} \dfrac{x(x+1)}{x+1} = \lim\limits_{x\to -1} x = -1$ ✓

3.) $2 \neq -1$ ✗

Not Continuous at $c=-1$.

d.) $f(x) = \begin{cases} \dfrac{x^2-4}{x^2-5x-14} & \text{if } x < 7 \\ 2\sqrt{x+2}-4 & \text{if } x \geq 7 \end{cases}, \qquad c = 7$

1.) $f(7) = 2\sqrt{7+2}-4 = 2\sqrt{9}-4 = 2(3)-4 = 2$ ✓

2.) $\lim\limits_{x\to 7^-} \dfrac{x^2-4}{x^2-5x-14} = \lim\limits_{x\to 7^-} \dfrac{(x-2)(x+2)}{(x+2)(x-7)} = \lim\limits_{x\to 7^-} \dfrac{x-2}{x-7} = -\infty$

$\lim\limits_{x\to 7^+} 2\sqrt{x+2}-4 = 2\sqrt{7+2}-4 = 2\sqrt{9}-4 = 2$ ✗

3.) N/A

Not Continuous at $c=7$.

e.) $f(x) = \begin{cases} \dfrac{x^2-4x+3}{x-3} & \text{if } x < 3 \\ 2 & \text{if } x = 3 \\ 2^{2x-3}-2x & \text{if } x > 3 \end{cases}, \qquad c = 3$

1.) $f(3) = 2$ ✓

2.) $\lim\limits_{x\to 3^-} \dfrac{x^2-4x+3}{x-3} = \lim\limits_{x\to 3^-} \dfrac{(x-3)(x-1)}{x-3} = \lim\limits_{x\to 3^-} x-1 = 2$ ✓

$\lim\limits_{x\to 3^+} 2^{2x-3}-2x = 2^{2(3)-3}-2(3) = 2^{6-3}-6 = 2^3-6 = 8-6 = 2$

3.) $2 = 2$ ✓

Continuous at $c=3$.

f.) $= \begin{cases} \sin(x) & \text{if } x < \frac{\pi}{2} \\ \tan\left(x-\frac{\pi}{4}\right) & \text{if } x \geq \frac{\pi}{2} \end{cases}, \qquad c = \dfrac{\pi}{2}$

1.) $f\left(\frac{\pi}{2}\right) = \tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$ ✓

2.) $\lim\limits_{x\to \frac{\pi}{2}^-} \sin(x) = \sin\left(\frac{\pi}{2}\right) = 1$ ✓

$\lim\limits_{x\to \frac{\pi}{2}^+} \tan\left(x-\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

3.) $1 = 1$ ✓

Continuous at $c=\frac{\pi}{2}$.