§2.4: Sum and Difference Formulas

1.] Find the exact value of $\sin(u+v)$ given that $\sin(u) = 4/5$, where u is an angle in the first quadrant, and $\cos(v) = -12/13$, where v is in the second quadrant.

Identity: $\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v)$

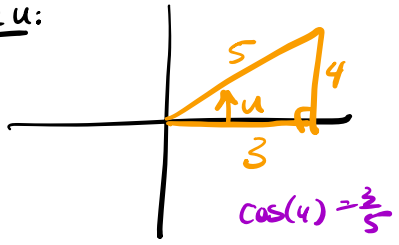
$= \frac{4}{5}\cdot\left(-\frac{12}{13}\right) + \cos(u)\sin(v)$

$= \frac{4}{5}\cdot\left(-\frac{12}{13}\right) + \frac{3}{5}\cdot\frac{5}{13}$

$= -\frac{48}{65} + \frac{15}{65}$

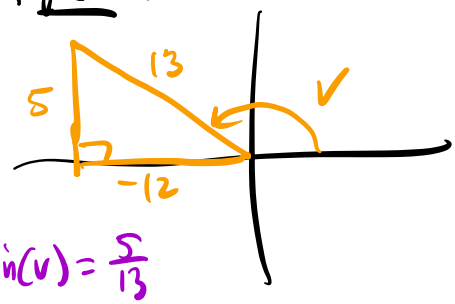
$= \boxed{-\frac{33}{65}}$

Angle u:



$\cos(u) = \frac{3}{5}$

Angle v:



$\sin(v) = \frac{5}{13}$

2.] Find all solutions to the following equation within the interval $[0, 2\pi)$:

$$\sin\left(x+\frac{\pi}{4}\right) + \sin\left(x-\frac{\pi}{4}\right) = -1$$

- $\sin\left(\overset{u}{x}+\overset{v}{\frac{\pi}{4}}\right) = \sin(x)\cos\left(\frac{\pi}{4}\right) + \cos(x)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\sin(x) + \frac{\sqrt{2}}{2}\cos(x)$
- $\sin\left(x-\frac{\pi}{4}\right) = \sin(x)\cos\left(\frac{\pi}{4}\right) - \cos(x)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\sin(x) - \frac{\sqrt{2}}{2}\cos(x)$
- Equation: $\sin\left(x+\frac{\pi}{4}\right) + \sin\left(x-\frac{\pi}{4}\right) = -1$

$$\frac{\sqrt{2}}{2}\sin(x) + \frac{\sqrt{2}}{2}\cos(x) + \frac{\sqrt{2}}{2}\sin(x) - \frac{\sqrt{2}}{2}\cos(x) = -1$$

$$\frac{\sqrt{2}}{2}\sin(x) + \cancel{\frac{\sqrt{2}}{2}\cos(x)} + \frac{\sqrt{2}}{2}\sin(x) - \cancel{\frac{\sqrt{2}}{2}\cos(x)} = -1$$

$$\sqrt{2}\sin(x) = -1$$

$$\sin(x) = -\frac{1}{\sqrt{2}}$$

$$\sin(x) = -\frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$$

3.] Find all solutions to the following equation within the interval $[0, 2\pi)$:

$$\tan(x+\pi) + 2\sin(x+\pi) = 0$$

- $\sin(x+\pi) = \sin(x)\cos(\pi) + \cos(x)\sin(\pi) = \sin(x)(-1) + \cos(x)(0)$
  $= -\sin(x)$
- $\tan(x+\pi) = \dfrac{\tan(x)+\tan(\pi)}{1-\tan(x)\tan(\pi)} = \dfrac{\tan(x)+0}{1-\tan(x)(0)} = \tan(x)$
- Substitute and solve: $\tan(x) + 2(-\sin(x)) = 0$

$\Rightarrow \tan(x) - 2\sin(x) = 0$

$\Rightarrow \dfrac{\sin(x)}{\cos(x)} - 2\sin(x) = 0$

$\Rightarrow \sin(x)\left(\dfrac{1}{\cos(x)} - 2\right) = 0$

$\sin(x) = 0$

$x = 0, \pi$

$\dfrac{1}{\cos(x)} - 2 = 0$

$\dfrac{1}{\cos(x)} = 2$

$\cos(x) = \dfrac{1}{2}$

$x = \pi/3, 5\pi/3$