§2.4 (PART 2): OTHER IMPORTANT LIMITS

1.] Important limits to know:

Limits at Infinity:

a.) $\lim\limits_{x\to\infty} \frac{1}{x} \approx \frac{1}{1{,}000{,}000} = .000001 \to 0$

$\boxed{\lim\limits_{x\to\infty} \frac{1}{x} = 0}$

b.) $\lim\limits_{x\to-\infty} \frac{1}{x} \approx \frac{1}{-1{,}000{,}000} = -.000001 \to 0$

$\boxed{\lim\limits_{x\to-\infty} \frac{1}{x} = 0}$

c.) $\lim\limits_{x\to\infty} x^2 - 2x - 8$

$= \lim\limits_{x\to\infty} x^2\left(1 - \frac{2}{x} - \frac{8}{x^2}\right) = \infty^2(1-0-0)$

$= \boxed{\infty}$

d.) $\lim\limits_{x\to-\infty} x^2 - 2x - 8$

$= \lim\limits_{x\to-\infty} x^2\left(1 - \frac{2}{x} - \frac{8}{x^2}\right) = (-\infty)^2\cdot(1-0-0)$

$= \boxed{\infty}$

e.) $\lim\limits_{x\to\infty} \frac{1}{x^2-2x-8}$

$= \lim\limits_{x\to\infty} \frac{1}{x^2(1-2/x-8/x^2)} = \frac{1}{\infty^2(1-0-0)} = \boxed{0}$

f.) $\lim\limits_{x\to\infty} \frac{x+3}{x^2-2x-8}$

$= \lim\limits_{x\to\infty} \frac{x(1+3/x)}{x^2(1-2/x-8/x^2)} = \lim\limits_{x\to\infty} \frac{1}{x}\left(\frac{1+3/x}{1-2/x-8/x^2}\right)$

$= \frac{1}{\infty}\left(\frac{1+0}{1-0-0}\right) = \boxed{0}$

g.) $\lim\limits_{x\to-\infty} \frac{x+3}{x^2-2x-8}$

$= \lim\limits_{x\to\infty} \frac{x(1+3/x)}{x^2(1-2/x-8/x^2)} = \lim\limits_{x\to\infty} \frac{1}{x}\left(\frac{1+3/x}{1-2/x-8/x^2}\right)$

$= \frac{1}{-\infty}\left(\frac{1+0}{1-0-0}\right) = \boxed{0}$

h.) $\lim\limits_{x\to\infty} \frac{x^2+3}{x^2-2x-8}$

$= \lim\limits_{x\to\infty} \frac{x^2(1+3/x)}{x^2(1-2/x-8/x^2)}$

$= \lim\limits_{x\to\infty} \frac{1+3/x}{1-2/x-8/x^2}$

$= \frac{1+0}{1-0-0} = \boxed{1}$

Infinite Limits:

a.) $\lim\limits_{x\to0^+} \frac{1}{x} \approx \frac{1}{.000001} = 1{,}000{,}000 \to \infty$

$\boxed{\lim\limits_{x\to0^+} \frac{1}{x} = \infty}$

b.) $\lim\limits_{x\to0^-} \frac{1}{x} \approx \frac{1}{-.000001} = -1{,}000{,}000 \to -\infty$

$\boxed{\lim\limits_{x\to0^-} \frac{1}{x} = -\infty}$

c.) $\lim\limits_{x\to4^-} \frac{10}{x^2-2x-8} = \lim\limits_{x\to4^-} \frac{10}{(x-4)(x+2)}$

$\approx \frac{10}{(-.0001)(6)} = \boxed{-\infty}$

d.) $\lim\limits_{x\to4^+} \frac{10}{x^2-2x-8} = \lim\limits_{x\to4^+} \frac{10}{(x-4)(x+2)}$

$\approx \frac{10}{(.0001)(6)} = \boxed{\infty}$

e.) $\lim\limits_{x\to-2^-} \frac{x}{x^2-2x-8} = \lim\limits_{x\to-2^-} \frac{x}{(x-4)(x+2)}$

$\approx \frac{-2}{(-6)(-.0001)} = \boxed{-\infty}$

f.) $\lim\limits_{x\to-2^+} \frac{2x+3}{x^2-2x-8} = \lim\limits_{x\to-2^+} \frac{2x+3}{(x-4)(x+2)}$

$\approx \frac{-1}{(-6)(.0001)} = \boxed{\infty}$

g.) $\lim\limits_{x\to-2^+} \frac{x-4}{x^2-2x-8} = \lim\limits_{x\to-2^+} \frac{\cancel{x-4}}{\cancel{(x-4)}(x+2)}$

$= \lim\limits_{x\to-2^+} \frac{1}{x+2} \approx \frac{1}{.0001} = \boxed{\infty}$

h.) $\lim\limits_{x\to-2^+} \frac{x+2}{x^2-2x-8}$

$= \lim\limits_{x\to-2^+} \frac{\cancel{x+2}}{\cancel{(x+2)}(x-4)}$

$= \lim\limits_{-2^+} \frac{1}{x-4} = \frac{1}{-2-4} = \boxed{-\frac{1}{6}}$

Note: In all three examples below, we obtain "$\frac{0}{0}$" when the limit value is plugged in. The value at the point is undefined but the limit still exists!

2.] Evaluate the following limits using algebra:

a.) $\lim\limits_{x\to 2} \dfrac{x^4-16}{3x^2-5x-2} = \lim\limits_{x\to 2} \dfrac{(x^2-4)(x^2+4)}{(3x+1)(x-2)}$

$$= \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}(x^2+4)}{(3x+1)\cancel{(x-2)}}$$

$$= \lim_{x\to 2} \frac{(x+2)(x^2+4)}{3x+1}$$

$$= \frac{(2+2)(2^2+4)}{3(2)+1}$$

$$= \frac{(4)(8)}{7}$$

$$= \boxed{\frac{32}{7}}$$

b.) $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7} = \lim\limits_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7}\left(\dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3}\right)$

$$= \lim_{x\to 7} \frac{(x+2)-9}{(x-7)(\sqrt{x+2}+3)}$$

$$= \lim_{x\to 7} \frac{\cancel{x-7}}{\cancel{(x-7)}(\sqrt{x+2}+3)}$$

$$= \lim_{x\to 7} \frac{1}{\sqrt{x+2}+3}$$

$$= \frac{1}{\sqrt{7+2}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \boxed{\frac{1}{6}}$$

c.) $\lim\limits_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$, where x is any real number except 0.

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{x-x-h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$= -\frac{1}{x(x+0)}$$

$$= \boxed{-\frac{1}{x^2}}$$