§2.4 (part 1): Determining Limits of Functions

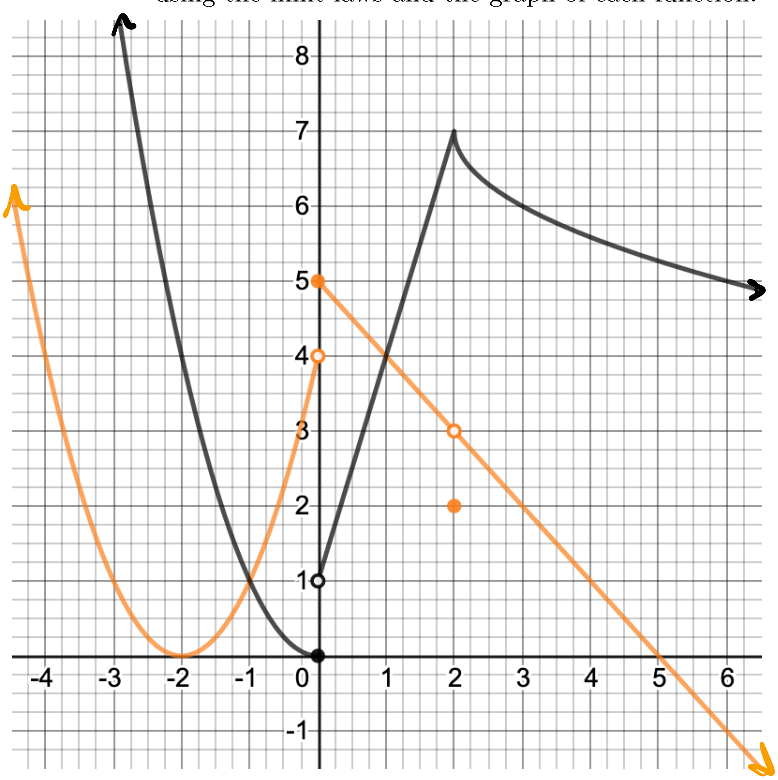
1.] Assuming $\lim_{x\to1} f(x) = 8$, $\lim_{x\to1} g(x) = 3$, and $\lim_{x\to1} h(x) = 2$, compute the following limits:

a.) $\lim_{x\to1} (f(x) - g(x)) = \lim_{x\to1} f(x) - \lim_{x\to1} g(x) = 8-3 = \boxed{5}$

b.) $\lim_{x\to1} \left[\frac{f(x)}{g(x)-h(x)}\right] = \frac{\lim_{x\to1} f(x)}{\lim_{x\to1} g(x) - \lim_{x\to1} h(x)} = \frac{8}{3-2} = \frac{8}{1} = \boxed{8}$

c.) $\lim_{x\to1} \sqrt[3]{f(x)g(x)+3} = \lim_{x\to1}(f(x)g(x)+3)^{1/3} = \left(\lim_{x\to1} f(x) \cdot \lim_{x\to1} g(x) + 3\right)^{1/3} = (8\cdot3+3)^{1/3} = 27^{1/3} = \boxed{3}$

2.] Consider the graph of $f(x)$ (black) and $g(x)$ (orange) below. Compute the following limits, if possible, using the limit laws and the graph of each function.



a.) $\lim_{x\to2} \left[\frac{f(x)}{2g(x)}\right] = \frac{\lim_{x\to2} f(x)}{2\cdot \lim_{x\to2} g(x)} = \frac{7}{2\cdot3} = \boxed{\frac{7}{6}}$

b.) $\lim_{x\to2} \left[\frac{3}{7}f(x) + g(x)\right] = \frac{3}{7}\cdot \lim_{x\to2} f(x) + \lim_{x\to2} g(x)$
$= \frac{3}{7}(7) + 3 = 3+3 = \boxed{6}$

c.) $\lim_{x\to0^+} [f(x)g(x)] = \left(\lim_{x\to0^+} f(x)\right)\cdot\left(\lim_{x\to0^+} g(x)\right) = (1)(5) = \boxed{5}$

d.) $\lim_{x\to0^-} [f(x)g(x)] = \left(\lim_{x\to0^-} f(x)\right)\cdot\left(\lim_{x\to0^-} g(x)\right) = (0)(4) = \boxed{0}$

e.) $\lim_{x\to0} [f(x)g(x)] =$ DNE (because the one-sided limits are not equal)

Use the graph above to compute the limit: $\lim_{x\to0} [g(x) - f(x)]$

Must compute one-sided limits:

$\lim_{x\to0^-} [g(x)-f(x)] = \lim_{x\to0^-} g(x) - \lim_{x\to0^-} f(x) = 5-1 = 4$

$\lim_{x\to0^+} [g(x)-f(x)] = \lim_{x\to0^+} g(x) - \lim_{x\to0^+} f(x) = 4-0 = 4$

Thus, $\lim_{x\to0} [g(x)-f(x)] = \boxed{4}$

3.] Compute the following limit: $\lim_{x\to -1} \left(2x^4 + x^3 - 3x^2 + 7\right)$

*This is a polynomial with domain $\mathbb{R}$. Just plug in -1!

$= 2(-1)^4 + (-1)^3 - 3(-1)^2 + 7$

$= 2(1) + (-1) - 3(1) + 7$

$= 2 - 1 - 3 + 7$

$= \boxed{5}$

4.] Compute the following limit: $\lim_{x\to 1} \dfrac{2x^3 - 5x + 1}{2x - x^2}$

*This is a rational function with domain $(-\infty,0)\cup(0,2)\cup(2,\infty)$. Just plug in 1!

$$= \frac{2(1)^3 - 5(1) + 1}{2(1) - (1)^2} = \frac{2-5+1}{2-1} = \frac{-2}{1} = \boxed{-2}$$

5.] Determine the following limits and provide a sketch of the graph of the function.

$f(x) = \dfrac{x^2-1}{x-1}$ Domain: $x \neq 1$

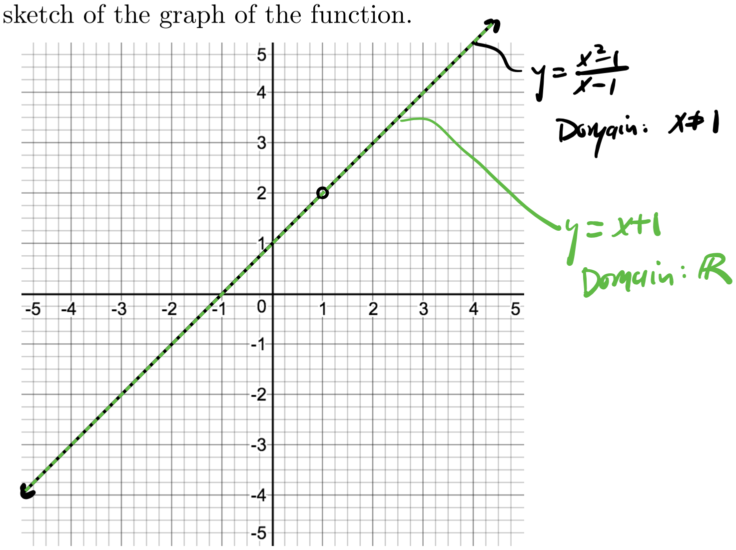
a.) $\lim_{x\to 1} \dfrac{x^2-1}{x-1} = \lim_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$

$= \lim_{x\to 1} x+1$

$= 1+1$

$= 2$

Note: $\lim_{x\to 1} f(x) = 2$, $f(1) = \text{DNE}$ $\to$ Hole at $(1,2)$



b.) $\lim_{x\to 9} \dfrac{9-x}{3-\sqrt{x}} = \lim_{x\to 9} \dfrac{9-x}{3-\sqrt{x}}\left(\dfrac{3+\sqrt{x}}{3+\sqrt{x}}\right)$

$= \lim_{x\to 9} \dfrac{\cancel{(9-x)}(3+\sqrt{x})}{\cancel{9-x}}$

$= \lim_{x\to 9} 3+\sqrt{x}$

$= 3+\sqrt{9}$

$= 6$

$f(x) = \dfrac{9-x}{3-\sqrt{x}}$

Domain: $[0,9)\cup(9,\infty)$

Note: $\lim_{x\to 9} f(x) = 6$, $f(9) = \text{DNE}$ $\to$ Hole at $(9,6)$