§2.3 (part 2): Solving Trigonometric Equations

1.] Suppose x is any angle inside $[0, 2\pi)$. Solve the following equation for x: $2\sin^2(x) - \sin(x) - 1 = 0$

$$2\sin^2(x) - \sin(x) - 1 = 0$$

$$\Rightarrow (2\sin(x)+1)(\sin(x)-1) = 0$$

$$\Rightarrow 2\sin(x)+1=0 \qquad \sin(x)-1=0$$

$$\Rightarrow 2\sin(x) = -1 \qquad \sin(x)=1$$

$$\sin(x) = -\tfrac{1}{2} \qquad \sin(x)=1$$

$$x = \tfrac{7\pi}{6}, \tfrac{11\pi}{6} \qquad x = \tfrac{\pi}{2}$$

$$\Rightarrow \boxed{x = \tfrac{\pi}{2}, \tfrac{7\pi}{6}, \tfrac{11\pi}{6}}$$

2.] Suppose x is any angle inside $[0, 2\pi)$. Solve the following equation for x: $2\sin^2(x) + 3\cos(x) = 3$

$$2\sin^2(x) + 3\cos(x) = 3$$

$$\Rightarrow 2(1-\cos^2(x)) + 3\cos(x) = 3$$

$$\Rightarrow 2 - 2\cos^2(x) + 3\cos(x) = 3$$

$$\Rightarrow 2\cos^2(x) - 3\cos(x) + 1 = 0$$

$$(2\cos(x)-1)(\cos(x)-1) = 0$$

$$\Rightarrow 2\cos(x)-1=0 \qquad \cos(x)-1=0$$

$$\Rightarrow \cos(x) = \tfrac{1}{2} \qquad \cos(x)=1$$

$$\Rightarrow x = \tfrac{\pi}{3}, \tfrac{5\pi}{3} \qquad x=0$$

$$\Rightarrow \boxed{x = 0, \tfrac{\pi}{3}, \tfrac{5\pi}{3}}$$

3.] Find the general solution to the equation: $\sin(x) + 1 = \cos(x)$

Beware of extraneous sols

$$\sin(x)+1 = \cos(x)$$

$$\Rightarrow (\sin(x)+1)^2 = (\cos(x))^2 \quad *$$

$$\Rightarrow \sin^2(x) + 2\sin(x) + 1 = \cos^2(x)$$

$$\Rightarrow \sin^2(x) + 2\sin(x) + 1 = 1 - \sin^2(x)$$

$$\Rightarrow 2\sin^2(x) + 2\sin(x) = 0$$

$$2\sin(x)(\sin(x)+1) = 0$$

$$2\sin(x)=0 \qquad \sin(x)+1=0$$

$$\sin(x)=0 \qquad \sin(x)=-1$$

$$x = 0, \pi \qquad x = 3\pi/2$$

Extraneous solution (doesn't satisfy original eqn)

$$\Rightarrow \boxed{x = 0 + 2n\pi, \ \tfrac{3\pi}{2} + 2n\pi}$$

4.] Find the general solution to the equation: $2\cos(3x) - 1 = 0$

$2\cos(3x) - 1 = 0$

$\Rightarrow 2\cos(3x) = 1$

$\Rightarrow \cos(3x) = \frac{1}{2}$

$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\rightarrow 3x = \frac{\pi}{3} + 2n\pi \qquad 3x = \frac{5\pi}{3} + 2n\pi$

$\Rightarrow \boxed{x = \frac{\pi}{9} + \frac{2}{3}n\pi \qquad x = \frac{5\pi}{9} + \frac{2}{3}n\pi}$

5.] Find the general solution to the equation: $2\tan\left(\frac{x}{2}\right) - 2 = 0$

$2\tan\left(\frac{x}{2}\right) - 2 = 0$

$\Rightarrow 2\tan\left(\frac{x}{2}\right) = 2$

$\Rightarrow \tan\left(\frac{x}{2}\right) = 1$

$\Rightarrow \frac{x}{2} = \frac{\pi}{4}, \frac{5\pi}{4}$

$\rightarrow \frac{x}{2} = \frac{\pi}{4} + 2n\pi, \quad \frac{x}{2} = \frac{5\pi}{4} + 2n\pi$

$\Rightarrow x = \frac{\pi}{2} + 4n\pi, \quad x = \frac{5\pi}{2} + 4n\pi$

$\boxed{x = \frac{\pi}{2} + 2n\pi}$

6.] Find all solutions in the interval $[0, 2\pi)$: $\sin^2(x) - 3\sin(x) - 2 = 0$

Doesn't factor nicely!

$\sin^2(x) - 3\sin(x) - 2 = 0$

$\Rightarrow \sin(x) = \frac{-(-3)}{2} \pm \frac{\sqrt{(-3)^2 - 4(1)(-2)}}{2}$

$\Rightarrow \sin(x) = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$

$\Rightarrow \sin(x) = \frac{3+\sqrt{17}}{2} \qquad \sin(x) = \frac{3-\sqrt{17}}{2}$

$\rightarrow x = \arcsin\left(\frac{3+\sqrt{17}}{2}\right), \; x = \arcsin\left(\frac{3-\sqrt{17}}{2}\right)$

$x = N/A \qquad x = -.596$

QIV answer

$x = \overbrace{2\pi - .596}^{\text{QIV}}, \; \overbrace{\pi + .596}^{\text{QIII}}$

$\boxed{x = 5.687, \; 3.738}$