

§2.3 (part 2): Solving Trigonometric Equations

1.] Suppose  $x$  is any angle inside  $[0, 2\pi)$ . Solve the following equation for  $x$ :  $2\sin^2(x) - \sin(x) - 1 = 0$

$$\begin{aligned}
 & 2\sin^2(x) - \sin(x) - 1 = 0 \\
 \Rightarrow & (2\sin(x) + 1)(\sin(x) - 1) = 0 \\
 \Rightarrow & 2\sin(x) + 1 = 0 \quad \sin(x) - 1 = 0 \\
 \Rightarrow & 2\sin(x) = -1 \quad \sin(x) = 1
 \end{aligned}$$

$$\begin{aligned}
 & \sin(x) = -\frac{1}{2} \quad \sin(x) = 1 \\
 & x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = \frac{\pi}{2} \\
 \Rightarrow & \boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}
 \end{aligned}$$

2.] Suppose  $x$  is any angle inside  $[0, 2\pi)$ . Solve the following equation for  $x$ :  $2\sin^2(x) + 3\cos(x) = 3$

$$\begin{aligned}
 & 2\sin^2(x) + 3\cos(x) = 3 \\
 \Rightarrow & 2(1 - \cos^2(x)) + 3\cos(x) = 3 \\
 \Rightarrow & 2 - 2\cos^2(x) + 3\cos(x) = 3 \\
 \Rightarrow & 2\cos^2(x) - 3\cos(x) + 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 & (2\cos(x) - 1)(\cos(x) - 1) = 0 \\
 \Rightarrow & 2\cos(x) - 1 = 0 \quad \cos(x) - 1 = 0 \\
 \Rightarrow & \cos(x) = \frac{1}{2} \quad \cos(x) = 1 \\
 \Rightarrow & x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = 0 \\
 \Rightarrow & \boxed{x = 0, \frac{\pi}{3}, \frac{5\pi}{3}}
 \end{aligned}$$

3.] Find the general solution to the equation:  $\sin(x) + 1 = \cos(x)$

*Be aware of extraneous solutions*

$$\begin{aligned}
 & \sin(x) + 1 = \cos(x) \\
 \Rightarrow & (\sin(x) + 1)^2 = (\cos(x))^2 \\
 \Rightarrow & \sin^2(x) + 2\sin(x) + 1 = \cos^2(x) \\
 \Rightarrow & \sin^2(x) + 2\sin(x) + 1 = 1 - \sin^2(x) \\
 \Rightarrow & 2\sin^2(x) + 2\sin(x) = 0
 \end{aligned}$$

$$\begin{aligned}
 & 2\sin(x)(\sin(x) + 1) = 0 \\
 & 2\sin(x) = 0 \quad \sin(x) + 1 = 0 \\
 & \sin(x) = 0 \quad \sin(x) = -1 \\
 & x = 0, \pi \quad x = \frac{3\pi}{2}
 \end{aligned}$$

*Extraneous solution (doesn't satisfy original eqn)*

$$\Rightarrow \boxed{x = 0 + 2n\pi, \frac{3\pi}{2} + 2n\pi}$$

4.] Find the general solution to the equation:  $2\cos(3x) - 1 = 0$ 

$$\begin{aligned} 2\cos(3x) - 1 &= 0 \\ \Rightarrow 2\cos(3x) &= 1 \\ \Rightarrow \cos(3x) &= \frac{1}{2} \\ \Rightarrow 3x &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

$$\Rightarrow \begin{aligned} 3x &= \frac{\pi}{3} + 2n\pi & 3x &= \frac{5\pi}{3} + 2n\pi \\ \Rightarrow \boxed{x &= \frac{\pi}{9} + \frac{2}{3}n\pi & x &= \frac{5\pi}{9} + \frac{2}{3}n\pi} \end{aligned}$$

5.] Find the general solution to the equation:  $2\tan\left(\frac{x}{2}\right) - 2 = 0$ 

$$\begin{aligned} 2\tan\left(\frac{x}{2}\right) - 2 &= 0 \\ \Rightarrow 2\tan\left(\frac{x}{2}\right) &= 2 \\ \Rightarrow \tan\left(\frac{x}{2}\right) &= 1 \\ \Rightarrow \frac{x}{2} &= \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$

$$\Rightarrow \begin{aligned} \frac{x}{2} &= \frac{\pi}{4} + 2n\pi & \frac{x}{2} &= \frac{5\pi}{4} + 2n\pi \\ \Rightarrow x &= \frac{\pi}{2} + 4n\pi & x &= \frac{5\pi}{2} + 4n\pi \end{aligned}$$

$$\Rightarrow \boxed{x = \frac{\pi}{2} + 2n\pi}$$

6.] Find all solutions in the interval  $[0, 2\pi)$ :  $\sin^2(x) - 3\sin(x) - 2 = 0$ *Doesn't factor nicely!*

$$\begin{aligned} \Rightarrow \sin^2(x) - 3\sin(x) - 2 &= 0 \\ \Rightarrow \sin(x) &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2} \\ \Rightarrow \sin(x) &= \frac{3 \pm \sqrt{17}}{2} \\ \Rightarrow \sin(x) &= \frac{3+\sqrt{17}}{2} & \sin(x) &= \frac{3-\sqrt{17}}{2} \end{aligned}$$

$$\begin{aligned} X &= \arcsin\left(\frac{3+\sqrt{17}}{2}\right), X = \arcsin\left(\frac{3-\sqrt{17}}{2}\right) \\ X &= \text{N/A} & X &= -.596 \\ & & & \text{QIV answer} \\ & & & \text{QIV} & \text{QIII} \\ & & & X = 2\pi - .596 & , \pi + .596 \end{aligned}$$

$$\Rightarrow \boxed{X = 5.687, 3.738}$$