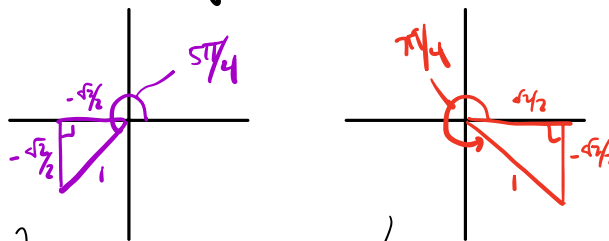


§2.3 (part 1): Solving Trigonometric Equations

1.] Suppose  $x$  is any angle inside  $[0, 2\pi)$ . Solve the following equation for  $x$ :  $\sin(x) + \sqrt{2} = -\sin(x)$

$$\begin{aligned} \sin(x) + \sqrt{2} &= -\sin(x) \\ \Rightarrow 2\sin(x) + \sqrt{2} &= 0 \\ \Rightarrow 2\sin(x) &= -\sqrt{2} \\ \Rightarrow \sin(x) &= -\frac{\sqrt{2}}{2} \end{aligned}$$

$\sin(x)$  is negative in QIII and QIV



$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

2.] Suppose  $x$  is any angle inside  $[0, 2\pi)$ . Solve the following equation for  $x$ :  $\cot(x) \cos^2(x) = 2 \cot(x)$

$$\begin{aligned} \cot(x) \cos^2(x) &= 2 \cot(x) \\ \Rightarrow \cot(x) \cos^2(x) - 2 \cot(x) &= 0 \\ \Rightarrow \cot(x) (\cos^2(x) - 2) &= 0 \\ \Rightarrow \cot(x) = 0 \quad \cos^2(x) - 2 &= 0 \end{aligned}$$

$\cot(x) = 0$

$$x = \pi/2, 3\pi/2$$

$\cos(x) = \pm\sqrt{2}$

$x = \text{N/A}$  ( $\cos(x)$  is never bigger than 1 or smaller than -1)

Because  $\cot(x)$  is zero at these values.

3.] Suppose  $x$  is any angle. Solve the following equation for  $x$ .  $\sin^2(x) = 2 \sin(x)$

$$\begin{aligned} \sin^2(x) &= 2 \sin(x) \\ \Rightarrow \sin^2(x) - 2 \sin(x) &= 0 \\ \Rightarrow \sin(x) (\sin(x) - 2) &= 0 \\ \Rightarrow \sin(x) = 0 \quad \sin(x) - 2 &= 0 \end{aligned}$$

$\sin(x) = 0$

$x = 0, \pi$

$\sin(x) = 2$

$x = \text{N/A}$  ( $\sin(x)$  is never 2)

$x = 0 + 2n\pi, \pi + 2n\pi$

or  $x = n\pi$  for any integer  $n$

4.] Find the general solution to the equation:  $3\sec^2(x) - 4 = 0$

$$\begin{aligned}
 & 3\sec^2(x) - 4 = 0 \\
 \Rightarrow & 3\sec^2(x) = 4 \\
 \Rightarrow & \sec^2(x) = \frac{4}{3} \\
 \Rightarrow & \sec(x) = \pm\sqrt{\frac{4}{3}}
 \end{aligned}$$

$$\begin{aligned}
 & \sec(x) = \frac{2}{\sqrt{3}} & \sec(x) = -\frac{2}{\sqrt{3}} \\
 & \cos(x) = \frac{\sqrt{3}}{2} & \cos(x) = -\frac{\sqrt{3}}{2} \\
 & x = \frac{\pi}{6}, \frac{11\pi}{6} & x = \frac{5\pi}{6}, \frac{7\pi}{6}
 \end{aligned}$$

$$\Rightarrow x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$$

5.] Find all solutions in the interval  $[0, 2\pi)$ :  $\sin^2(x) = 3\cos^2(x)$

$$\begin{aligned}
 & \sin^2(x) = 3\cos^2(x) \\
 \Rightarrow & 1 - \cos^2(x) = 3\cos^2(x) \\
 \Rightarrow & 1 = 4\cos^2(x) \\
 \Rightarrow & 4\cos^2(x) = 1 \\
 \Rightarrow & \cos^2(x) = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 & \cos(x) = \pm\frac{1}{2} \\
 & \cos(x) = \frac{1}{2} & \cos(x) = -\frac{1}{2} \\
 & x = \frac{\pi}{3}, \frac{5\pi}{3} & x = \frac{2\pi}{3}, \frac{4\pi}{3}
 \end{aligned}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

6.] Find all solutions in the interval  $[0, 2\pi)$ :  $\tan^2(x) = \sec(x) - 1$

$$\begin{aligned}
 & \tan^2(x) = \sec(x) - 1 \\
 \Rightarrow & \sec^2(x) - 1 = \sec(x) - 1 \\
 \Rightarrow & \sec^2(x) - \sec(x) = 0 \\
 \Rightarrow & \sec(x)(\sec(x) - 1) = 0 \\
 \Rightarrow & \sec(x) = 0 \quad \sec(x) - 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 & \sec(x) = 0 & \sec(x) = 1 \\
 & x = \text{N/A} & \Rightarrow \cos(x) = 1 \\
 & (\sec(x) \text{ has no } x\text{-intercepts}) & \Rightarrow x = 0
 \end{aligned}$$

$$\Rightarrow x = 0$$