

§2.2: Verifying Trigonometric Identities

1.] Verify the identity: $\frac{\sec^2(\theta) - 1}{\sec^2(\theta)} = \sin^2(\theta)$

LHS RHS

LHS: $\frac{\sec^2(\theta) - 1}{\sec^2(\theta)} = \frac{\sec^2(\theta)}{\sec^2(\theta)} - \frac{1}{\sec^2(\theta)}$

Identity: $1 = \sin^2(\theta) + \cos^2(\theta)$

$$= 1 - \cos^2(\theta)$$

$$= \sin^2(\theta) + \cos^2(\theta) - \cos^2(\theta)$$

$$= \sin^2(\theta) \quad \text{: RHS}$$

2.] Verify the identity: $2 \csc^2(\beta) = \frac{1}{1 - \cos(\beta)} + \frac{1}{1 + \cos(\beta)}$

LHS RHS

RHS: $\frac{1}{1 - \cos(\beta)} + \frac{1}{1 + \cos(\beta)} = \frac{1}{1 - \cos(\beta)} \left(\frac{1 + \cos(\beta)}{1 + \cos(\beta)} \right) + \frac{1}{1 + \cos(\beta)} \left(\frac{1 - \cos(\beta)}{1 - \cos(\beta)} \right)$

Common denominator

$$= \frac{1 + \cos(\beta)}{1 - \cos^2(\beta)} + \frac{1 - \cos(\beta)}{1 - \cos^2(\beta)}$$

$\sin^2(\beta) = 1 - \cos^2(\beta)$

$$= \frac{1 + \cos(\beta) + 1 - \cos(\beta)}{\sin^2(\beta)} = \frac{2}{\sin^2(\beta)} = 2 \csc^2(\beta) \quad \text{: LHS}$$

3.] Verify the identity: $(\tan^2(x) + 1)(\cos^2(x) - 1) = -\tan^2(x)$

LHS RHS

LHS: $(\tan^2(x) + 1)(\cos^2(x) - 1) = \sec^2(x)(-\sin^2(x))$

$$= -\frac{\sin^2(x)}{\cos^2(x)}$$

$$= -\tan^2(x) \quad \text{: RHS}$$

$\sec^2(x) = \tan^2(x) + 1$
 $\cos^2(x) - 1 = -\sin^2(x)$

4.] Verify the identity: $\tan(x) + \cot(x) = \sec(x) \csc(x)$

LHS RHS

$$\text{LHS: } \tan(x) + \cot(x) = \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}$$

Common denominator

$$= \frac{\sin(x)}{\cos(x)} \left(\frac{\sin(x)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left(\frac{\cos(x)}{\cos(x)} \right)$$

$$= \frac{\sin^2(x) + \cos^2(x)}{\cos(x)\sin(x)}$$

$\cos^2(x) + \sin^2(x) = 1$

$$= \frac{1}{\cos(x)\sin(x)} = \sec(x)\csc(x) : \text{RHS}$$

5.] Verify the identity: $\sec(x) + \tan(x) = \frac{\cos(x)}{1 - \sin(x)}$

LHS RHS

$$\text{RHS: } \frac{\cos(x)}{1 - \sin(x)} = \frac{\cos(x)}{1 - \sin(x)} \left(\frac{1 + \sin(x)}{1 + \sin(x)} \right)$$

$$= \frac{\cos(x)(1 + \sin(x))}{1 - \sin^2(x)}$$

$1 - \sin^2(x) = \cos^2(x)$

$$= \frac{\cos(x)(1 + \sin(x))}{\cos^2(x)}$$

$$= \frac{1 + \sin(x)}{\cos(x)}$$

$$= \frac{1}{\cos(x)} + \frac{\sin(x)}{\cos(x)}$$

$$= \sec(x) + \tan(x) : \text{LHS}$$

6.] Verify the identity: $\tan^4(x) = \tan^2(x) \sec^2(x) - \tan^2(x)$

LHS RHS

$$\text{RHS: } \tan^2(x) \sec^2(x) - \tan^2(x) = \tan^2(x) (\sec^2(x) - 1)$$

$$= \tan^2(x) \tan^2(x)$$

$\sec^2(x) - 1 = \tan^2(x)$

$$= \tan^4(x) : \text{LHS}$$