§2.2 (part 2): Limits All Around the Plane

1.] Let $f(x) = \dfrac{5x+6}{x-2}$. Evaluate the limits $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$ without using the graph but by evaluating the function directly using a computer algebra system (e.g. Desmos) or a calculator. Fill out the following tables below.

x	-10	-100	-1000	-10000
$f(x)$	3.666667	4.84314	4.98403	4.99840

$$\lim_{x\to-\infty} \frac{5x+6}{x-2} = 5$$

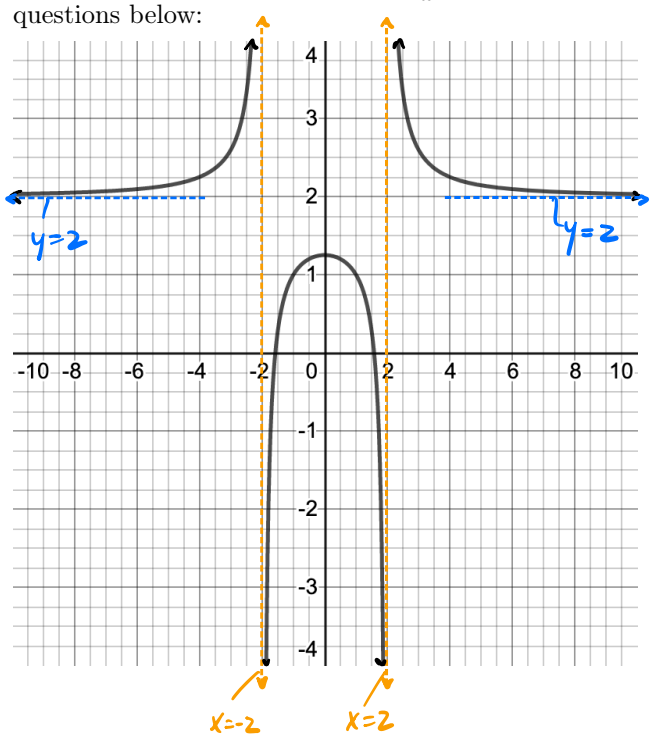
x	10	100	1000	10000
$f(x)$	7	5.16327	5.01603	5.00160

$$\lim_{x\to\infty} \frac{5x+6}{x-2} = 5$$

Deduce whether or not the function has a horizontal asymptote.

Horizontal Asymptote at $y=5$

2.] The graph of the function $f(x) = \dfrac{2x^2-5}{x^2-4}$ is shown below. Is $f(x)$ algebraic or transcendental? Answer questions below:



a.) Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

b.) $\lim\limits_{x\to-2^-} f(x) = \infty \longrightarrow$ VA at $x=-2$

c.) $\lim\limits_{x\to-2^+} f(x) = -\infty \longrightarrow$ VA at $x=-2$

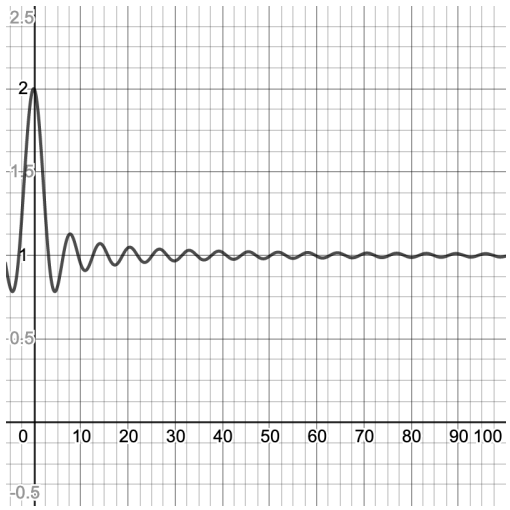
d.) $\lim\limits_{x\to2^-} f(x) = -\infty \longrightarrow$ VA at $x=2$

e.) $\lim\limits_{x\to2^+} f(x) = \infty \longrightarrow$ VA at $x=2$

f.) $\lim\limits_{x\to-\infty} f(x) = 2 \longrightarrow$ HA at $y=2$

g.) $\lim\limits_{x\to\infty} f(x) = 2 \longrightarrow$ HA at $y=2$

3.] Consider the graph of the function $f(x) = 1 + \dfrac{\sin(x)}{x}$ below.



* Note: the amplitude is steadily decreasing and the values of $f(x)$ are getting closer to 1 as $x \to \infty$.

Does this function have a horizontal asymptote? To answer this question, evaluate the following limit:

$\lim_{x\to\infty} 1 + \dfrac{\sin(x)}{x} \approx 1 + \dfrac{\sin(1{,}000{,}000)}{1{,}000{,}000}$ Squeeze Theorem

$1 + \dfrac{-1}{1{,}000{,}000} \le 1 + \dfrac{\sin(1{,}000{,}000)}{1{,}000{,}000} \le 1 + \dfrac{1}{1{,}000{,}000}$

$0.999999 \le 1 + \dfrac{\sin(1{,}000{,}000)}{1{,}000{,}000} \le 1.000001$

$\Rightarrow 1 \le \lim_{x\to\infty} 1 + \dfrac{\sin(x)}{x} \le 1$

$\Rightarrow \lim_{x\to\infty} 1 + \dfrac{\sin(x)}{x} = 1$

↳ HA at $y = 1$

4.] Determine the end behavior of the function $f(x) = x^2 - 2x - 8$.

$\lim_{x\to-\infty} x^2 - 2x - 8 \approx (-1000)^2 - 2(-1000) - 8$
$= 1000000 + 2000 - 8 \to \infty$

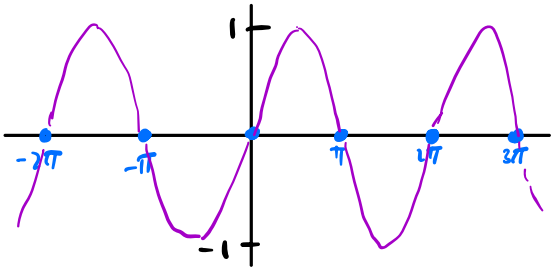
$\lim_{x\to\infty} x^2 - 2x - 8 \approx (1000)^2 - 2(1000) - 8$
$= 1000000 - 2000 - 8 \to \infty$

$\lim_{x\to-\infty} x^2 - 2x - 8 = \infty$

$\lim_{x\to\infty} x^2 - 2x - 8 = \infty$

↳ There are no horizontal asymptotes.

5.] Determine if the function $f(x) = \sin(x)$ has a horizontal asymptote.



$\lim_{x\to-\infty} \sin(x) = \text{DNE}$

$\lim_{x\to\infty} \sin(x) = \text{DNE}$

↳ The function values continue to oscillate and never settle to a finite number