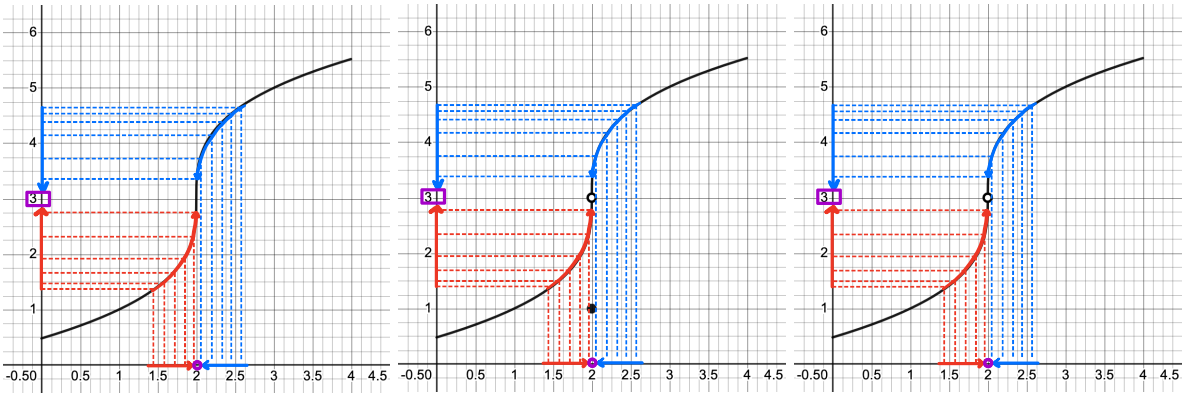
§2.2 (part 1): Limits All Around the Plane

1.] Given the graph of the functions below, determine the limits:

$$f(x) = 2\sqrt[3]{x-2}+3 \qquad g(x) = \begin{cases} 2\sqrt[3]{x-2}+3 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases} \qquad h(x) = \begin{cases} 2\sqrt[3]{x-2}+3 & \text{if } x \neq 2 \\ \text{undefined} & \text{if } x = 2 \end{cases}$$



a.) $\lim_{x\to 2} f(x) = 3$

b.) $f(2) = 3$

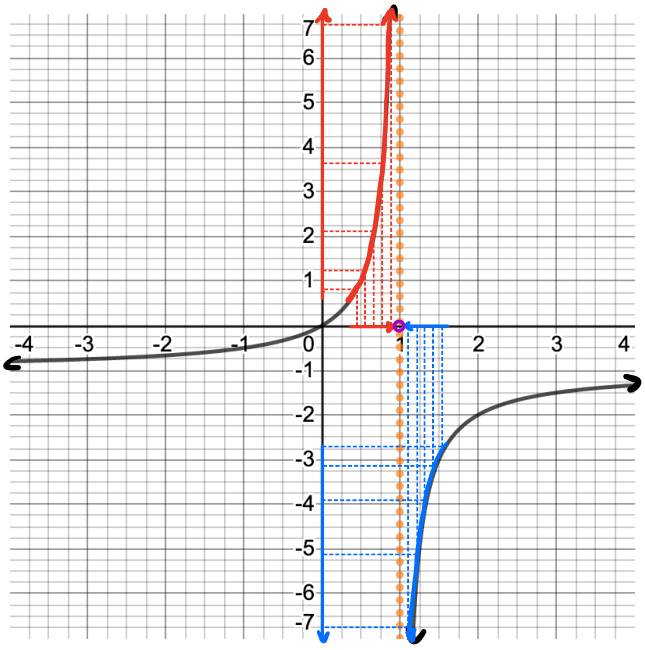
c.) $\lim_{x\to 2} g(x) = 3$

d.) $g(2) = 1$

e.) $\lim_{x\to 2} h(x) = 3$

f.) $h(2) =$ DNE

2.] The graph of the function $f(x) = \dfrac{-x}{x-1}$ is shown below. Is $f(x)$ algebraic or transcendental? Answer questions below:



a.) Domain: $(-\infty, 1) \cup (1, \infty)$

b.) Range: $(-\infty, -1) \cup (-1, \infty)$

c.) f is decreasing on N/A

d.) f is increasing on its domain.

e.) $\lim_{x\to 2} f(x) = -2$

f.) $f(2) = -2$

g.) $\lim_{x\to 1^-} f(x) = \infty$ (VA at $x=1$)

h.) $\lim_{x\to 1^+} f(x) = -\infty$ (VA at $x=1$)

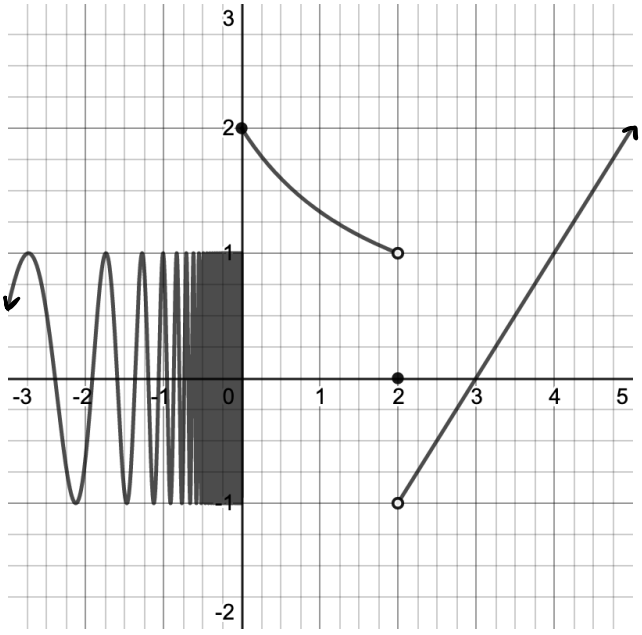
i.) $\lim_{x\to 1} f(x) =$ DNE

j.) $f(1) =$ DNE (not in domain)

3.] The graph of the piecewise function

$$f(x) = \begin{cases} \sin\left(\dfrac{30}{x}\right) & \text{if } x < 0 \\ \dfrac{4}{x+2} & \text{if } 0 \le x < 2 \\ 0 & \text{if } x = 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

is given below. Evaluate the limits:



a.) $\lim_{x\to 0^-} f(x)$ = DNE (oscillates)

b.) $\lim_{x\to 0^+} f(x)$ = 2

c.) $\lim_{x\to 0} f(x)$ = DNE

d.) $f(0)$ = 2

e.) $\lim_{x\to 2^-} f(x)$ = 1

f.) $\lim_{x\to 2^+} f(x)$ = −1

g.) $\lim_{x\to 2} f(x)$ = DNE

h.) $f(2)$ = 0

4.] Let $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$. Evaluate the limits $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ without using the graph but by evaluating the function directly using a computer algebra system (e.g. Desmos) or a calculator. Fill out the following tables below.

x	1.9	1.99	1.999	1.9999
$f(x)$	−1.1	−1.01	−1.001	−1.0001

$$\lim_{x\to 2^-} \frac{x^2 - 5x + 6}{x - 2} = -1$$

x	2.1	2.01	2.001	2.0001
$f(x)$	−0.9	−0.99	−0.999	−0.9999

$$\lim_{x\to 2^+} \frac{x^2 - 5x + 6}{x - 2} = -1$$

Using the evidence gathered above, deduce the value of the following limit, if it exists:

$$\lim_{x\to 2} \frac{x^2 - 5x + 6}{x - 2} = -1$$