§2.1: Rates of Change and Tangents

1.] Suppose a grenade is launched vertically upwards from the ground with a speed of 96 ft/s. Neglecting air resistance, a well-known formula from physics states that the position of the grenade after t seconds is given by the function

$$d(t) = -16t^2 + 96t.$$

Find the average velocity of the grenade between 1 and 3 seconds of it being in the air.

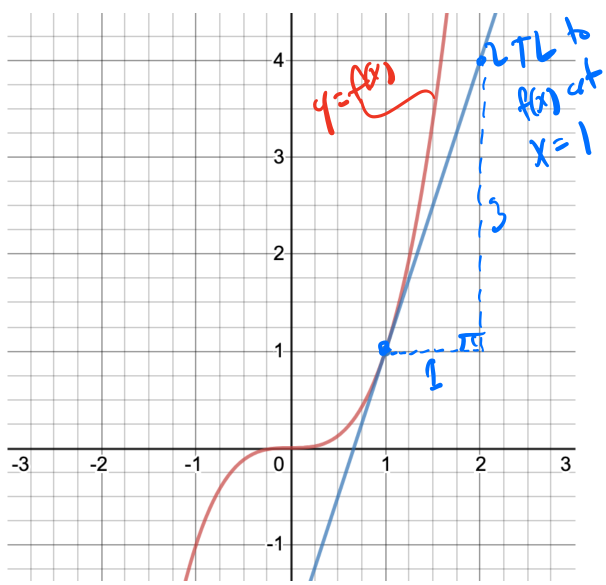
$$[\text{Avg Velocity}] = \frac{d(3)-d(1)}{3-1} = \frac{144-80}{2} = \frac{64}{2} = 32 \text{ ft/sec}$$

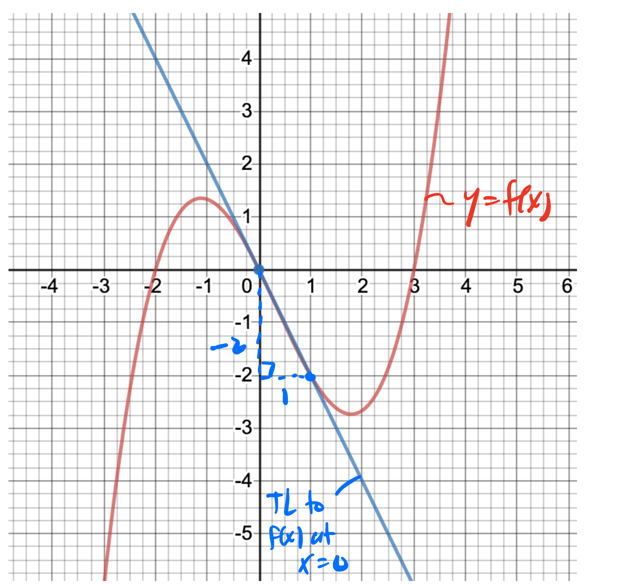
Over the time interval $t=1$ to $t=3$, the grenade is travelling, on average, 32 ft/sec.

$$d(3) = -16(3)^2 + 96(3) = 3(-16(3)+96) = 3(-48+96) = 3(48) = 144$$

$$d(1) = -16(1)^2 + 96(1) = -16+96 = 80$$

2.] Estimate the slope of the tangent line shown in the given graphs below:





- Slope of the tangent line to $f(x)$ at the point $(1,1)$ is $m=3$
- Instanteneous rate of change of $f(x)$ at $x=1$ is 3.
- $f'(1) = \lim_{x \to 1} \frac{f(x)-f(1)}{x-1} = 3.$

- Slope of the tangent line to $f(x)$ at the point $(0,0)$ is $m=-2$
- Instanteneous rate of change of $f(x)$ at $x=0$ is -2.
- $f'(0) = \lim_{x \to 0} \frac{f(x)-f(0)}{x-0} = -2$

3.] Consider the position function $d(t) = 16t^2$, where $d(t)$ is the distance a piece of rock has fallen from a 256-foot-deep canyon, if we ignore air resistance. Here, d is measured in feet and t is measured in seconds. Estimate the instantaneous velocity of the rock after two seconds.

$\lim\limits_{x \to 0^+} \frac{\Delta d}{\Delta t}$

Time interval	a b [2, 2.5]	a b [2, 2.1]	a b [2, 2.01]	a b [2, 2.001]	
Change in time (Δt) $b-a$	.5	.1	.01	.001	
Change in distance (Δd) $d(b)-d(a)$	36	6.56	.06416	.064016	
Average velocity $\left(\frac{\Delta d}{\Delta t}\right)$ $\frac{d(b)-d(a)}{b-a}$	72	65.6	64.16	64.016	$\to 64$

$\lim\limits_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$

$\lim\limits_{\Delta t \to 0^-} \frac{\Delta d}{\Delta t}$

Time interval	b a [1.5, 2]	b a [1.9, 2]	b a [1.99, 2]	b a [1.999, 2]	
Change in time (Δt)	−.5	−.1	−.01	−.001	
Change in distance (Δd)	−28	−6.24	−.6384	−.063984	
Average velocity $\left(\frac{\Delta d}{\Delta t}\right)$	56	62.4	63.84	63.984	$\to 64$

Make a conjecture about the value of the instantaneous velocity at $t = 2$.

From the brute force method above, we can conclude

$$\lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t} = \lim_{b \to a} \frac{d(b)-d(a)}{b-a} = 64 \text{ ft/sec}$$

The rock, after falling for 2 seconds, is traveling at an instantaneous velocity of 64 ft/sec.