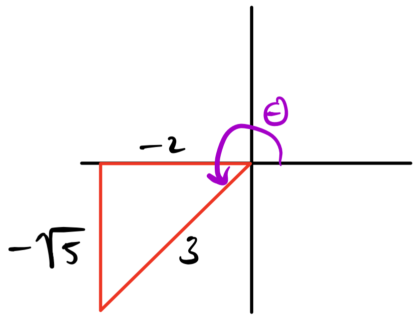
§2.1: Using Fundamental Identities

1.] Suppose we know that $\sec(\theta) = -\frac{3}{2}$ and $\tan(\theta) > 0$. Use appropriate identities to find all six ratios of this angle θ.

we know $\cos(\theta) = -\frac{2}{3} < 0$ and $\tan(\theta) > 0$, therefore θ is in Quadrant III and adj $= -2$, hyp $= 3$.

$3^2 = y^2 + (-2)^2 \rightarrow y = -\sqrt{5}$



$\cos(\theta) = -\frac{2}{3}$ $\sec(\theta) = -\frac{3}{2}$

$\sin(\theta) = -\frac{\sqrt{5}}{3}$ $\csc(\theta) = -\frac{3}{\sqrt{5}}$

$\tan(\theta) = \frac{\sqrt{5}}{2}$ $\cot(\theta) = \frac{2}{\sqrt{5}}$

2.] Simplify the following trigonometric expression: $\sin(x)\cos^2(x) - \sin(x)$.

$\sin(x)\cos^2(x) - \sin(x)$

factor out $\sin(x)$

$\sin(x)\left(\cos^2(x) - 1\right)$

use Identity: $\sin^2(x) + \cos^2(x) = 1$

$\Rightarrow \cos^2(x) - 1 = -\sin^2(x)$

$\sin(x)\left(-\sin^2(x)\right)$

$\Rightarrow \boxed{-\sin^3(x)}$

3.] Factor the following trigonometric expression: $\tan^2(\theta) + 2\tan(\theta) - 3$

Note: $x^2 + 2x - 3$

$= (x-1)(x+3)$

same idea

$\tan^2(\theta) + 2\tan(\theta) - 3$

$= (\tan(\theta))^2 + 2\tan(\theta) - 3$

$= \boxed{(\tan(\theta) - 1)(\tan(\theta) + 3)}$

4.] Use an identities to factor the following trigonometric expression: $\csc^2(x) - \cot(x) - 3$

Note: Use the identity:

$1 + \cot^2(x) = \csc^2(x)$

$\csc^2(x) - \cot(x) - 3$

$= \left(1 + \cot^2(x)\right) - \cot(x) - 3$

$= \cot^2(x) - \cot(x) - 2$

$= \boxed{(\cot(x) + 1)(\cot(x) - 2)}$

5.] Perform addition on the following expression and then simplify: $\dfrac{\sin(\theta)}{1+\cos(\theta)} + \dfrac{\cos(\theta)}{\sin(\theta)}$

$$\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{\cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1+\cos(\theta)}\left(\frac{\sin(\theta)}{\sin(\theta)}\right) + \frac{\cos(\theta)}{\sin(\theta)}\left(\frac{1+\cos(\theta)}{1+\cos(\theta)}\right)$$

common denominator

$$= \frac{\sin^2(\theta)}{(1+\cos(\theta))\sin(\theta)} + \frac{\cos(\theta)+\cos^2(\theta)}{\sin(\theta)(1+\cos(\theta))}$$

Note: $\cos^2(\theta)+\sin^2(\theta)=1$

$$= \frac{\sin^2(\theta)+\cos(\theta)+\cos^2(\theta)}{(1+\cos(\theta))\sin(\theta)} = \frac{\cancel{1+\cos(\theta)}}{\cancel{(1+\cos(\theta))}\sin(\theta)}$$

$$= \frac{1}{\sin(\theta)} = \boxed{\csc(\theta)}$$

6.] Rewrite the following expression so it is not in fractional form: $\dfrac{1}{1+\sin(x)}$

$$\frac{1}{1+\sin(x)} = \frac{1}{1+\sin(x)}\cdot\left(\frac{1-\sin(x)}{1-\sin(x)}\right)$$

$$= \frac{1-\sin(x)}{(1+\sin(x))(1-\sin(x))}$$

$$= \frac{1-\sin(x)}{1-\sin^2(x)}$$

use $1-\sin^2(x)=\cos^2(x)$

$$= \frac{1-\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} - \frac{\sin(x)}{\cos^2(x)}$$

$$= \boxed{\sec^2(x) - \sin(x)\tan(x)}$$

7.] Use the substitution $x = 3\sin(\theta)$ to write $\sqrt{9-x^2}$ as a trigonometric function of θ.

$$\sqrt{9-x^2} = \sqrt{9-(3\sin(\theta))^2}$$

$$= \sqrt{9-9\sin^2(\theta)}$$

$$= \sqrt{9(1-\sin^2(\theta))}$$

Use $1-\sin^2(\theta)=\cos^2(\theta)$

$$= \sqrt{9\cos^2(\theta)}$$

$$= \boxed{3\cos(\theta)}$$