§11.2: Two-Person Constant-Sum Games: Mixed Strategies

1.] Two players, A and B, play the coin-tossing game. Each player, unbeknownst to teh other chooses a head (H) or a tail (T). Both players would reveal their choices simultaneously. If they match (HH or TT), player A receives \$1 from B. Otherwise, A pays B. Set up the reward matrix for player A and find the value of the game by considering mixed strategies from each player.

		y_1	y_2	
		B_H	B_T	Row Min
x_1	A_H	1	−1	−1
x_2	A_T	−1	1	−1
	Col Max	1	1	

maximin = −1

minimax = 1

V = value of the game

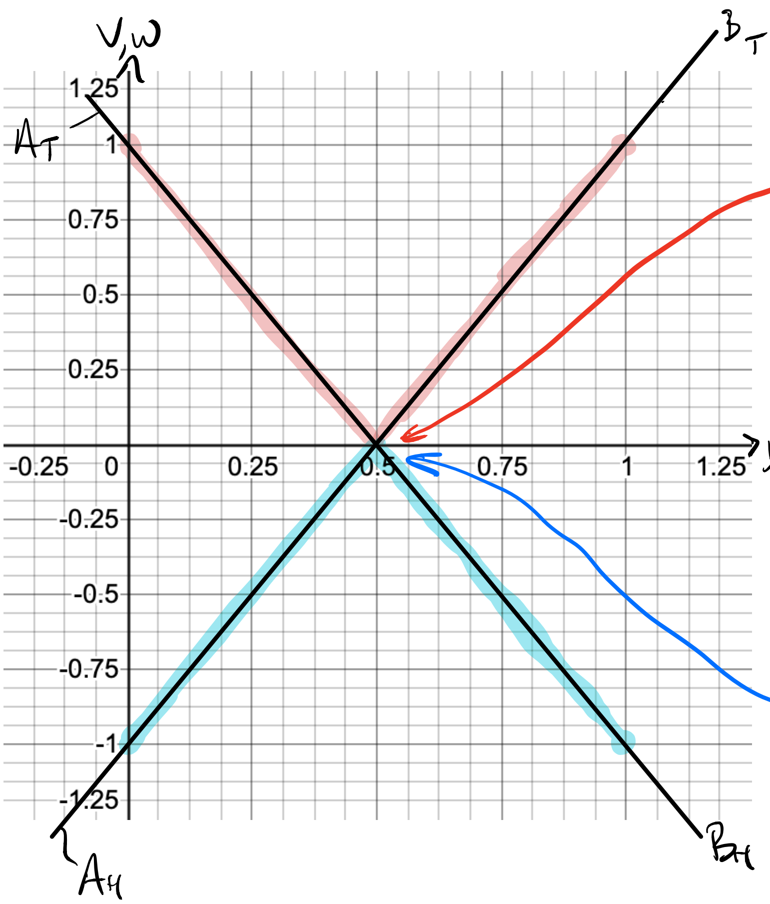
$$-1 \le V \le 1$$

B's Pure Strategy	A's Expected Payout
B_H	$x_1(1)+(1-x_1)(-1) = 2x_1-1$
B_T	$x_1(-1)+(1-x_1)(1) = 1-2x_1$

A's Pure Strategy	B Expected Payout
A_H	$y_1(1)+(1-y_1)(-1) = 2y_1-1$
A_T	$y_1(-1)+(1-y_1)(1) = 1-2y_1$

$x_2 = 1-x_1$

$y_2 = 1-y_1$



= B's greatest losses (col max's)

minimax = 0 occurs when $y_1 = 0.5$ and $y_2 = 1-0.5 = 0.5$

Thus, B should play B_H and B_T with probability 1/2.

= A's least gains (row min's)

maximin = 0 occurs when $x_1 = 0.5$ and $x_2 = 1-0.5 = 0.5$

Hence, A should play A_H and A_T with probability 1/2.

2.] Consider the following game where player A has two strategies and player B has four strategies. The reward matrix is in terms of payoff to player A. Determine the value of the game and the strategies employed by each player that results in the optimal saddle point solution.

		y_1	y_2	y_3	y_4	
		B_1	B_2	B_3	B_4	Row Min
x_1	A_1	2	2	3	-1	-1
$1-x_1$	A_2	4	3	2	6	2
	Col Max	4	3	3	6	

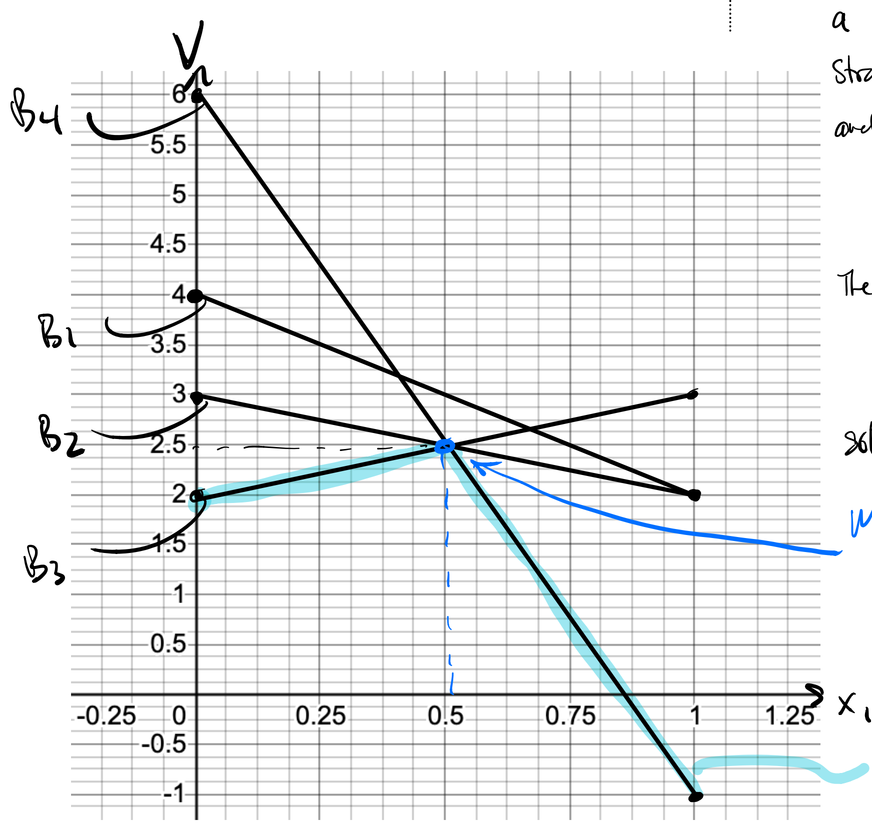
maximin = 2

minimax = 3

$2 \le V \le 3$

B's Pure Strategy	A's Expected Payoff
B_1	$2x_1 + 4(1-x_1) = -2x_1 + 4$
B_2	$2x_1 + 3(1-x_1) = -x_1 + 3$
B_3	$3x_1 + 2(1-x_1) = x_1 + 2$
B_4	$-x_1 + 6(1-x_1) = -7x_1 + 6$

*Since each expected value is a linear function of x_1, we can graph it below.



Maximum $= 2.5$ occurs when $x_1 = 0.5$.

Hence, A should play A_1 and A_2 with probabilities $\boxed{x_1 = 0.5 \text{ and } x_2 = 0.5}$.

= A's row minima

A's Pure Strategy	B's Expected Payoff
A_1	$2y_1 + 2y_2 + 3y_3 - y_4$
A_2	$4y_1 + 3y_2 + 2y_3 + 6y_4$

* we cannot graphically represent the two expected values. However, from A's graph we see that strategies B_3 and B_4 will yield the optimal value of the game, $v = 2.5$. Hence, a combination of y_3 and y_4 is an optimal strategy for B. Let $y_1 = y_2 = 0$, then $y_4 = 1 - y_3$ and

$$3y_3 - (1-y_3) = 4y_3 - 1$$

$$2y_3 + 6(1-y_3) = -4y_3 + 6$$

The intersection will yield the optimal strategy:

$$4y_3 - 1 = -4y_3 + 6 \Rightarrow \boxed{y_3 = \frac{7}{8},\ y_4 = \frac{1}{8}}$$

Solutions involving B_2 exist as well.

Value of the game: $\boxed{v = 2.5}$