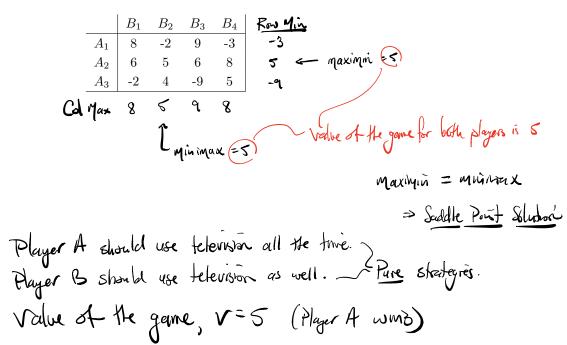## §11.1: Two-Person Constant-Sum Games: Saddle Points

1.] Two companies A and B sell two brands of Nintendonitis medication. Company A advertises in radio (A_1) televeision (A_2), and newspapers (A_3). Company B, in addition to using radio (B_1), television (B_2), and newspapers (B_4), also mails brochures (B_4). Depending on the effectiveness of each advertising campaign, one company can capture a portion of the market from the other. The following reward matrix summarizes the percentage of the market captured or lost by company A. Determine the optimal strategy for each company and find the value of the game.

	B_1	B_2	B_3	B_4	Row Min
A_1	8	-2	9	-3	-3
A_2	6	5	6	8	5 ← maximin =5
A_3	-2	4	-9	5	-9

Col Max   8   5   9   8

minimax =5

value of the game for both players is 5

Maximin = minimax

⇒ Saddle Point Solution

Player A should use television all the time.
Player B should use television as well. } Pure strategies.

Value of the game, $v = 5$ (Player A wins)

2.] Find the value and the optimal strategy for the two-person zero-sum game given by the following reward matrix:

	B_1	B_2	Row Min
A_1	2	2	2 ← Maximin = 2
A_2	1	3	1

Col Max   2   3

minimax = 2

value of the game for both players is 2.

Pure Saddle Point Solution: Player A uses strategy A_1 100% of the time
Player B uses strategy B_1 100% of the time
Value of the game, $v = 2$ (Player A wins)

3.] Find the value and the optimal strategy for the two-person zero-sum game given by the following reward matrix:

	B_1	B_2	B_3	B_4	Row Min
A_1	4	5	5	8	4
A_2	6	7	6	9	6 ← Maximin = 6
A_3	5	7	5	4	4
A_4	6	6	5	5	5

Col Max: 6   7   6   9

Minimax = 6

Value of the game for both players is 6.

Pure Saddle Point Solution : Player A uses strategy A_2 100% of the time
(with Alternative)   Player B uses strategy B_1 or B_3.
                            Value of the game, V=2 (Player A wins.)

4.] During the 8 to 9 PM time slot, two networks (A and B) are vying for an audience of 100 million viewers. The networks must simultaneously announce the type of show they will air in that time slot. The possible choices for each network and the number of network A viewers (in millions) for each choice are shown in the matrix below. This is a two-person constant-sum game. Does this game have a saddle point? What is the value of the game to network A and to network B?

Network B

		Western	Soap Opera	Comedy	Row Min
Network A	Western	35	15	60	15
	Soap Opera	45	58	50	45 ← Maximin = 45
	Comedy	28	14	70	14

Col Max   45   58   70

Minimax = 45

Value of the game is 45.

Pure Saddle Point Solution :• Network A should air the Soap Opera 100% of the time which will yield 45 million viewers.

• Network B should air the Western 100% of the time which will yield 100−45 = 55 million viewers.

• Value of the game, V=45 (Player B wins.)