§1.4: Trigonometric Functions of Any Angle
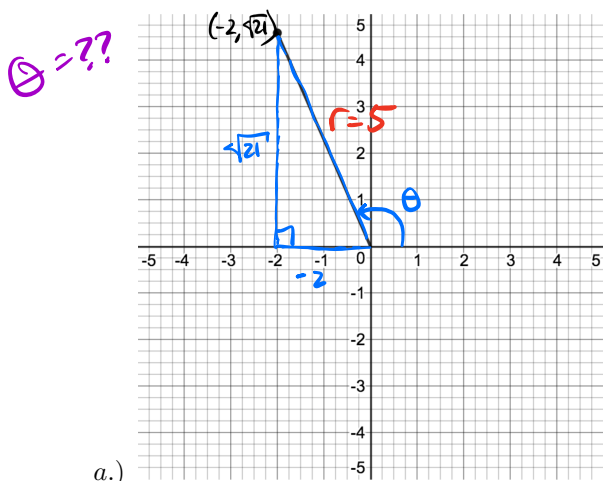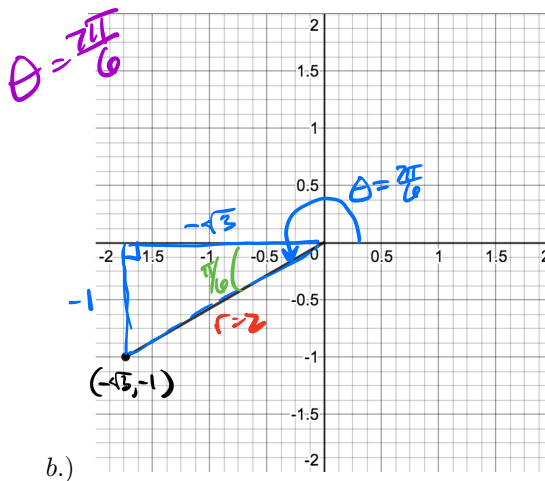
1.] Find the exact values of the six trigonometric functions of the angle θ below:

$\theta = ??$

$(-2, \sqrt{21})$

$r = 5$

$\sqrt{21}$

θ

-2

$\theta = \dfrac{7\pi}{6}$

$\theta = \dfrac{7\pi}{6}$

$-\sqrt{3}$

$\dfrac{7\pi}{6}$

$r = 2$

-1

$(-\sqrt{3}, -1)$

a.)

Pythagorean: $r^2 = \sqrt{21}^2 + (-2)^2 \rightarrow r^2 = 25$

$\Rightarrow r^2 = 21 + 4$ $r = 5$

$\cos(\theta) = -2/5$ $\sec(\theta) = -5/2$

$\sin(\theta) = \sqrt{21}/5$ $\csc(\theta) = 5/\sqrt{21}$

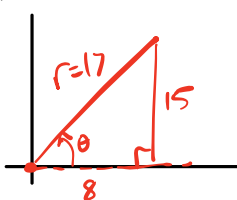$\tan(\theta) = -\sqrt{21}/2$ $\cot(\theta) = -2/\sqrt{21}$

b.)

Pythagorean: $r^2 = (-\sqrt{3})^2 + (-1)^2 \rightarrow r^2 = 4$

$\Rightarrow r^2 = 3 + 1$ $r = 2$

$\cos(\theta) = -\sqrt{3}/2$ $\sec(\theta) = -2/\sqrt{3}$

$\sin(\theta) = -1/2$ $\csc(\theta) = -2$

$\tan(\theta) = 1/\sqrt{3}$ $\cot(\theta) = \sqrt{3}$

2.] Suppose that $\tan(\theta) = \dfrac{15}{8}$ and $\sin(\theta) > 0$. Find the exact values of the remaining trigonometric functions of θ.
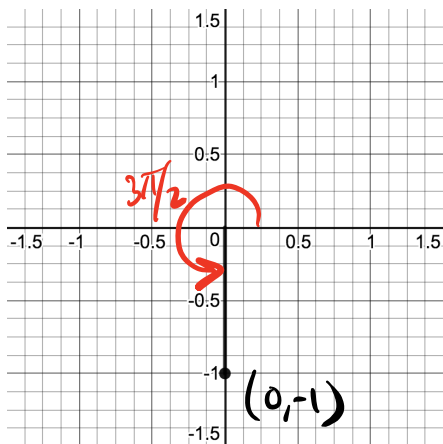
Note: Since $\tan(\theta) > 0$ and $\sin(\theta) > 0$, we know θ is in QI.

$r = 17$

15

θ

8

$r^2 = 15^2 + 8^2$

$r^2 = 225 + 64$

$r^2 = 289$

$r = 17$

$\cos(\theta) = 8/17$ $\sec(\theta) = 17/8$

$\sin(\theta) = 15/17$ $\csc(\theta) = 17/15$

$\tan(\theta) = 15/8$ $\cot(\theta) = 8/15$

3.] Evaluate the six trigonometric functions at $\theta = \dfrac{3\pi}{2}$.

$\dfrac{3\pi}{2}$

$(0, -1)$

$\cos\left(\dfrac{3\pi}{2}\right) = 0$

$\sin\left(\dfrac{3\pi}{2}\right) = -1$

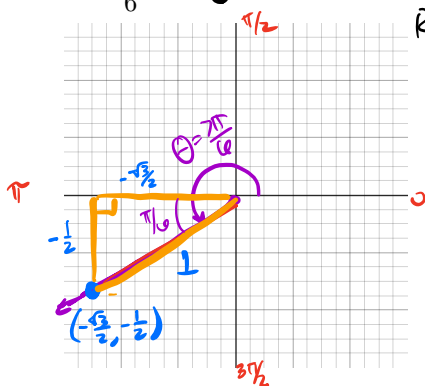$\tan\left(\dfrac{3\pi}{2}\right) = \dfrac{-1}{0} = $ undefined

$\sec\left(\dfrac{3\pi}{2}\right) = \dfrac{1}{0} = $ undefined

$\csc\left(\dfrac{3\pi}{2}\right) = \dfrac{1}{-1} = -1$

$\cot\left(\dfrac{3\pi}{2}\right) = \dfrac{0}{-1} = 0$

4.] For each angle θ below, find the reference angle θ' in both degrees and radians, and sketch the terminal side of θ.
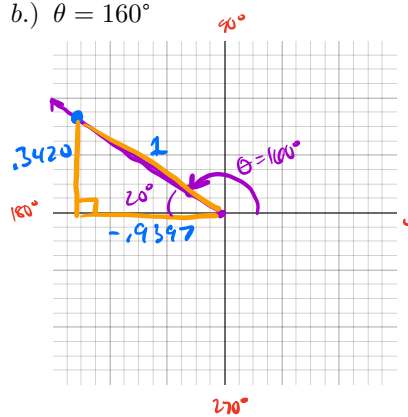
a.) $\theta = \dfrac{7\pi}{6} = 1\frac{1}{6}\pi$

Reference Angle
$\theta' = \frac{\pi}{6}$

$\cos\left(\frac{2\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{2\pi}{6}\right) = -\frac{1}{2}$

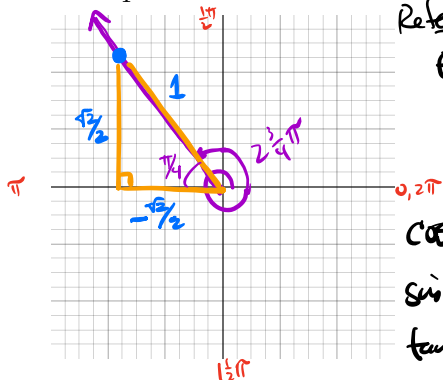$\tan\left(\frac{2\pi}{6}\right) = \frac{1}{\sqrt{3}}$

b.) $\theta = 160°$

Reference Angle
$\theta' = 20°$

From calculator

$\cos(160°) = -.4397$

$\sin(160°) = .3420$

$\tan(160°) = -.3640$

5.] For each angle below, evaluate the sine, cosine, and tangent of the angle by constructing the reference angle. (Do not use a calculator.)
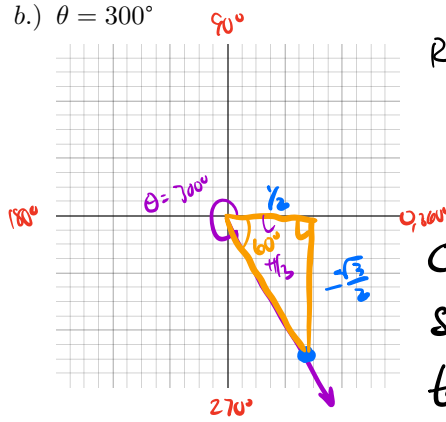
a.) $\theta = \dfrac{11\pi}{4} = 2\frac{3}{4}\pi \equiv \frac{3}{4}\pi$

Reference Angle
$\theta' = \frac{\pi}{4}$

$\cos\left(\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\sin\left(\frac{11\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\tan\left(\frac{11\pi}{4}\right) = -1$

b.) $\theta = 300°$
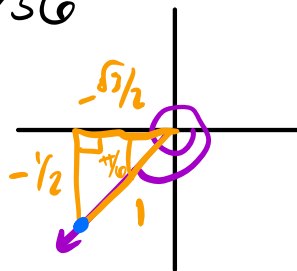
Reference Angle
$\theta' = 60°$

$\cos(300°) = \frac{1}{2}$

$\sin(300°) = -\frac{\sqrt{3}}{2}$

$\tan(300°) = -\sqrt{3}$

6.] Calculate the following trigonometric values. Find the exact value if possible; otherwise, use a calculator and round to four decimal places. Sketch the angle in every case.

a.) $\sin(10°)$ → need calculator

→ $\sin(10°) = .1736$

b.) $\cos\left(-\dfrac{17\pi}{6}\right)$ → $-\frac{17\pi}{6} = -2\frac{5}{6}\pi$ → $\cos\left(-\frac{17\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

c.) $\sec\left(\dfrac{11\pi}{8}\right)$ → need calculator

→ $\sec\left(\frac{11\pi}{8}\right) = \dfrac{1}{\cos\left(\frac{11\pi}{8}\right)} = -2.6131$