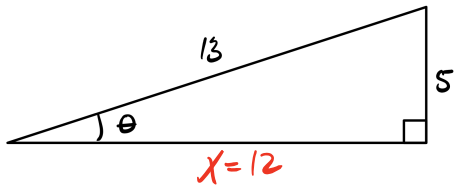
§1.3: Right Triangle Trigonometry

1.] Find the exact values of the six trigonometric functions of the angle θ below:

a.)



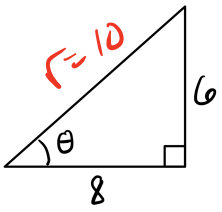
Pythagorean: $13^2 = x^2+5^2$ → $144 = x^2$, $x = 12$

$\cos(\theta) = 12/13$ $\quad \sec(\theta) = 13/12$

$\sin(\theta) = 5/13$ $\quad \csc(\theta) = 13/5$

$\tan(\theta) = 5/12$ $\quad \cot(\theta) = 12/5$

b.)



Pythagorean: $r^2 = 8^2+6^2$ → $r^2 = 100$, $r = 10$

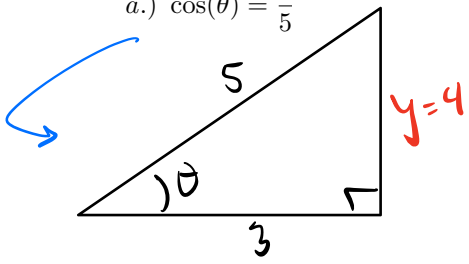
$\cos(\theta) = 8/10 = 4/5$ $\quad \sec(\theta) = 5/4$

$\sin(\theta) = 6/10 = 3/5$ $\quad \csc(\theta) = 5/3$

$\tan(\theta) = 6/8 = 3/4$ $\quad \cot(\theta) = 4/3$

2.] Sketch a right triangle corresponding trigonometric function of the acute angle θ. Then find the exact values of the other five trigonometric functions of θ.

a.) $\cos(\theta) = \frac{3}{5}$



Pythagorean: $5^2 = 3^2+y^2$ → $25 = 9+y^2$ → $y^2 = 16$, $y = 4$

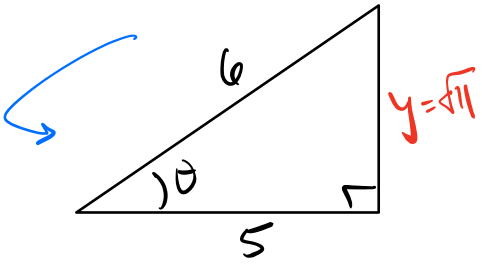
$\cos(\theta) = 8/10 = 4/5$ $\quad \sec(\theta) = 5/4$

$\sin(\theta) = 6/10 = 3/5$ $\quad \csc(\theta) = 5/3$

$\tan(\theta) = 6/8 = 3/4$ $\quad \cot(\theta) = 4/3$

b.) $\sec(\theta) = \frac{6}{5}$

$\Rightarrow \cos(\theta) = \frac{5}{6}$



Pythagorean: $6^2 = 5^2+y^2$ → $36 = 25+y^2$ → $y^2 = 11$, $y = \sqrt{11}$

$\cos(\theta) = 5/6$ $\quad \sec(\theta) = 6/5$

$\sin(\theta) = \sqrt{11}/6$ $\quad \csc(\theta) = 6/\sqrt{11}$

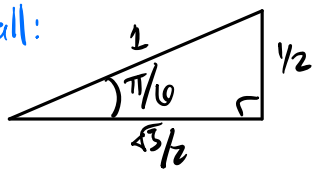
$\tan(\theta) = \sqrt{11}/5$ $\quad \cot(\theta) = 5/\sqrt{11}$

3.] Find the value of θ in radians without using a calculator:

a.) $\csc(\theta) = 2$

$\Rightarrow \sin(\theta) = \frac{1}{2}$

Recall:

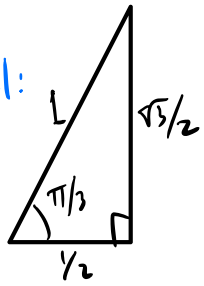


$\Rightarrow \boxed{\theta = \pi/6}$

b.) $\cos(\theta) = \frac{3}{6}$

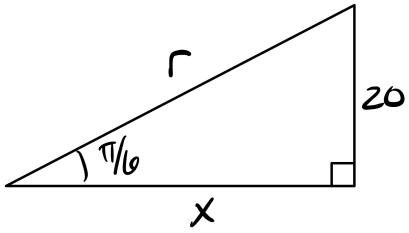
$\Rightarrow \cos(\theta) = \frac{1}{2}$

Recall:



$\Rightarrow \boxed{\theta = \pi/3}$

4.] Find the values of x and r on the triangle below:

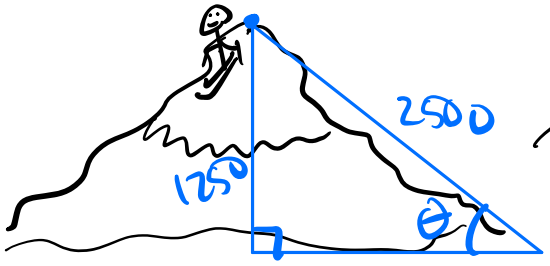


$\sin\left(\frac{\pi}{6}\right) = \frac{20}{r}$ $\quad \cos\left(\frac{\pi}{6}\right) = \frac{x}{r}$

$\Rightarrow \frac{1}{2} = \frac{20}{r}$ $\quad \Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{40}$

$\Rightarrow \boxed{r = 40}$ $\quad \Rightarrow \boxed{x = 20\sqrt{3}}$

5.] You are skiing down a mountain with vertical height of 1250 ft. The distance from the top of the mountain to the base is 2500 ft. What is the angle of elevation from the base to the top of the mountain?



$\sin(\theta) = \frac{1250}{2500}$

$\Rightarrow \sin(\theta) = \frac{1}{2} \Rightarrow \boxed{\theta = \frac{\pi}{6}}$

6.] Show, using the appropriate identities, that the equation $\sin^2(\theta) - \cos^2(\theta) = 2\sin^2(\theta) - 1$.

LHS: RHS

LHS: $\sin^2(\theta) - \cos^2(\theta) = \sin^2(\theta) - (1 - \sin^2(\theta))$ Used Identity

$= \sin^2(\theta) - 1 + \sin^2(\theta)$

$= 2\sin^2(\theta) - 1$: RHS ✓

Because $\sin^2(\theta) + \cos^2(\theta) = 1$, we know $\cos^2(\theta) = 1 - \sin^2(\theta)$.