§1.1: Prerequisites and Functions

1.] Find the sum:

$\frac{2\pi}{3} + \frac{3}{7}\pi = \frac{2\pi}{3} + \frac{3\pi}{7} = \frac{2\pi}{3}\left(\frac{7}{7}\right) + \frac{3\pi}{7}\left(\frac{3}{3}\right) = \frac{14\pi}{21} + \frac{9\pi}{21} = \boxed{\frac{23\pi}{21}}$

2.] Determine if each equation below is correct or incorrect:

a.) $\sqrt{2} + \sqrt{2} = 2$ Incorrect Correct: $\sqrt{2} + \sqrt{2} = 2\sqrt{2} = \sqrt{4}\sqrt{2} = \sqrt{4 \cdot 2} = \sqrt{8}$ or $\sqrt{2} \cdot \sqrt{2} = (\sqrt{2})^2 = 2$

b.) $(x-4)^2 = x^2 - 16$ Incorrect Correct: $(x-4)^2 = (x-4)(x-4) = x^2 - 4x - 4x + 16 = x^2 - 8x + 16$ or $x^2 - 16 = (x+4)(x-4)$

c.) $\sqrt{x^2+36} = x + 6$ Incorrect $\sqrt{x^2+36}$ cannot be simplified further.

d.) $e^{2x} = e^x \cdot e^x$ Correct! $e^{2x} = e^{x+x} = e^x \cdot e^x$ $e^{2x} = (e^x)^2 = e^x \cdot e^x$

e.) $\sin(x+\pi) = \sin(x) + \sin(\pi)$ Incorrect Correct: $\sin(x+\pi) = \sin(x)\cos(\pi) + \cos(x)\sin(\pi)$ (Trig Identity)

f.) $\sqrt{16x} = 4\sqrt{x}$ Correct $\sqrt{16x} = \sqrt{16} \cdot \sqrt{x} = 4\sqrt{x}$

3.] Simplify the following expressions so negative exponents and radical forms are eliminated:

a.) $\sqrt{3^5 + 3^5 + 3^5} = \sqrt{3 \cdot 3^5} = \sqrt{3^1 \cdot 3^5} = \sqrt{3^6} = (3^6)^{1/2} = 3^{6 \cdot \frac{1}{2}} = 3^3 = \boxed{27}$

b.) $\frac{4(x^2)^{-3}\sqrt{y^3}}{4^{-2}(x^{-1})^2 y^2} = \frac{4 \cdot 4^2 x^{-6} y^{3/2}}{x^{-2} y^2} = \frac{4^3}{x^{-2} x^6 y^2 y^{-3/2}} = \boxed{\frac{64}{x^4 y^{1/2}}}$

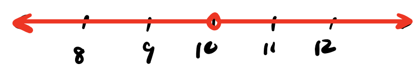
c.) $(x + y^{-1})^{-1} = \frac{1}{x + y^{-1}} = \frac{1}{x + \frac{1}{y}} = \frac{1}{x\left(\frac{y}{y}\right) + \frac{1}{y}} = \frac{1}{\frac{xy}{y} + \frac{1}{y}} = \frac{1}{\frac{xy+1}{y}} = \boxed{\frac{y}{xy+1}}$

4.] Find the implied domain of the function $f(x) = 4x^3 + 2^x - 5$.

Domain: $(-\infty, \infty)$ or $\mathbb{R}$

5.] Find the implied domain of the function $f(x) = \dfrac{1}{x-10}$.

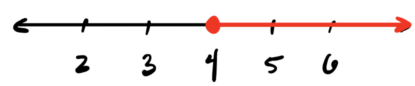
Domain: $(-\infty, 10) \cup (10, \infty)$



$x - 10 \neq 0 \rightarrow x \neq 10$

6.] Find the implied domain of the function $f(x) = \sqrt{x-4}$.

Domain: $[4, \infty)$



$x - 4 \geq 0 \rightarrow x \geq 4$

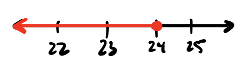
7.] Find the implied domain of the function $f(x) = \dfrac{\sqrt{24-x}}{x^2 - x - 6}$.

Num: $24 - x \geq 0$

$\Rightarrow 24 \geq x$

$\Rightarrow x \leq 24$

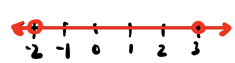
$(-\infty, 24]$



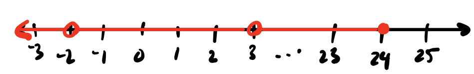
Den: $x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

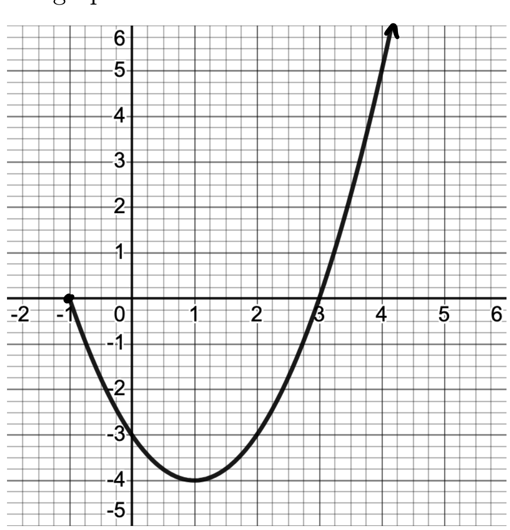
roots: $x = -2, 3 \Rightarrow x \neq -2, 3$



Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, 24]$



8.] Use the graph of $y = f(x)$ below to determine the domain and range, and find the function values using the graph.



a.) Domain: $[-1, \infty)$

b.) Range: $[-4, \infty)$

c.) $f(-2) =$ DNE (not in domain)

d.) $f(0) = -3$

e.) $f(2) = -3$

f.) f is decreasing on $(-1, 1)$

g.) f is increasing on $(1, \infty)$

h.) The zeros of $f(x)$ are $x = -1, 3$

i.) is f monotonic on its domain?

No, this function changes from decreasing to increasing at $x = 1$.