

Name: _____

Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Desmos graphs can be submitted separately.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. This assignment is graded on effort, completeness, and neatness for a total of 5 points. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer may result in a loss of points. Any parts to any questions that are not answered will also result in a loss of points. Academic dishonesty will not be tolerated.

PROBLEM SET VIII

MAT 181 – CALCULUS I

DUE: FRIDAY, MAY 3rd BY 11:59 PM ON D2L

READ: SECTIONS 5.2–5.4

1. Suppose $\int_1^3 f(x) dx = 5$ and $\int_1^3 g(x) dx = -11$. Evaluate the following, **if possible**:

(a) $\int_1^3 (2f(x) - 4g(x)) dx$

(b) $\int_3^1 3f(x) dx$

(c) $\int_3^1 \frac{f(x)}{1+f(x)} dx$

(d) $\int_1^2 f(x) dx + \int_2^3 f(x) dx$

(e) $\int_1^1 f(x)\sqrt{|g(x)|} dx$

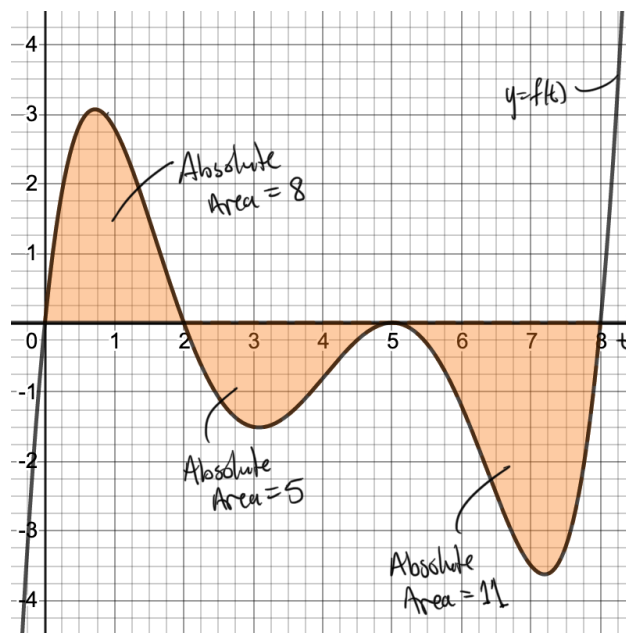
(f) $\int_1^3 f(x)g(x) dx$

(g) $\int_1^6 f(x) - 2g(x) dx$

(h) Find the average value of $f(x)$ over the interval $[1, 3]$.

2. Compute the area under the function $f(x) = 4x^2 + 3x + 2$ on the interval $[0, 3]$ exactly by evaluating the integral using the limit definition of a Riemann Sum using rectangles with right-hand endpoints as the sample points.

3. Suppose the following graph is of the function $y = f(t)$:



Consider two area functions defined by $A(x) = \int_0^x f(t) dt$ and $B(x) = \int_2^x f(t) dt$. Evaluate the following:

(a) $A(2)$

(b) $B(5)$

(c) $A(0)$

(d) $B(8)$

(e) $A(8)$

(f) $A(5)$

(g) $B(2)$

(h) $B(0)$

(i) $A'(2)$

4. (This question spans two pages.) Evaluate the following indefinite and definite integrals using the Fundamental Theorem of Calculus and u -substitution where necessary:

(a) $\int_{-2}^1 (x^2 - x - 6) dx$

(b) $\int_0^{\pi/4} 2 \cos(x) dx$

(c) $\int_0^4 10 \ln(2) 2^x dx$

(d) $\int_0^1 x \sqrt[3]{x} dx$

(e) $\int_1^{\sqrt{3}} \frac{8}{1+x^2} dx$

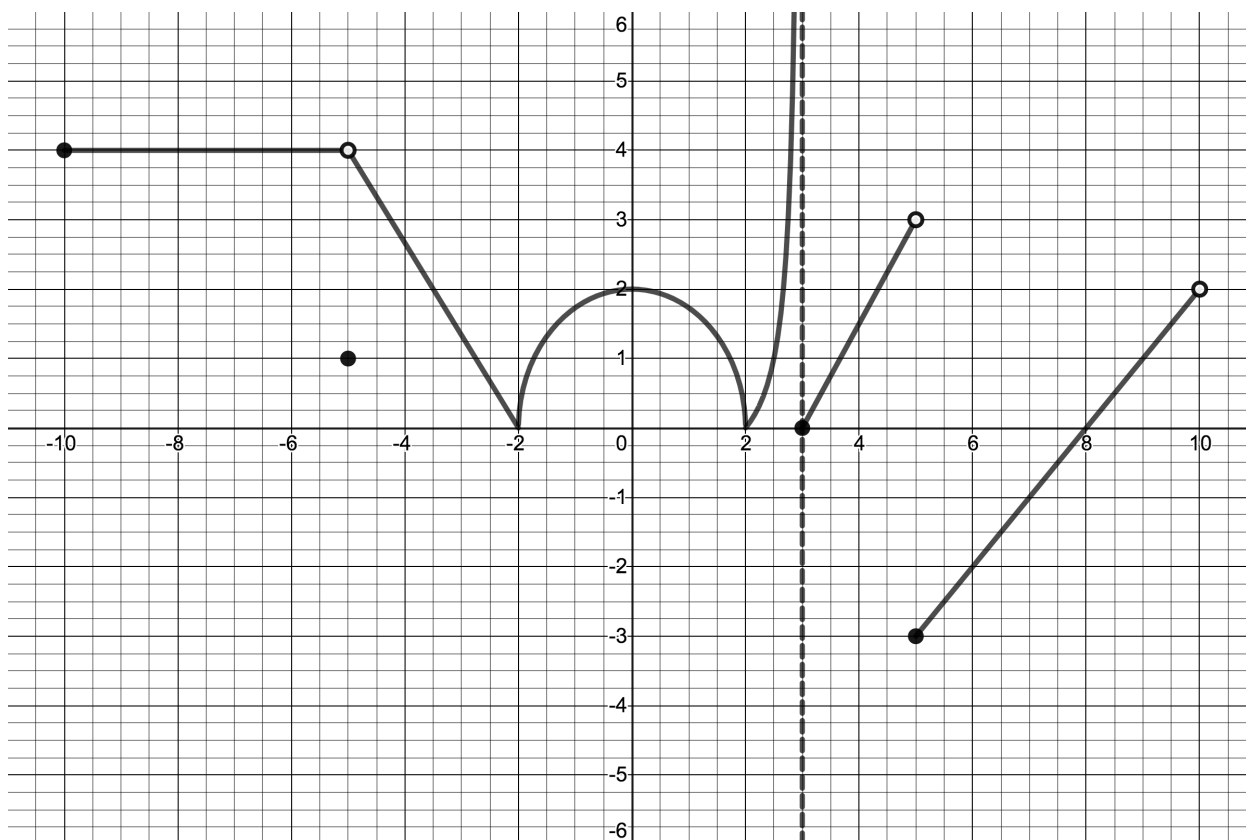
$$(f) \int 8x \cos(4x^2 + 3) dx$$

$$(g) \int \frac{2x}{4x^2 + 1} dx$$

$$(h) \int \frac{5 \arctan(x)}{1 + x^2} dx$$

$$(i) \int_0^4 \frac{x}{\sqrt{9 + x^2}} dx$$

5. Treat this question as a “One-Question Calculus Final Exam.” This question covers most topics that we discussed this semester. The function, $y = f(x)$, below is piecewise and the shape in the middle is a semi-circle. Answer all the questions that follow.



- (a) Compute $f'(4)$.
- (b) Compute $\int_{-10}^{-6} f(x) dx$.
- (c) Compute $\int_3^{10} |f(x)| dx$.
- (d) Does $f(x)$ satisfy Rolle’s Theorem on $[-2, 2]$? If not, say why. If so, find the value of c guaranteed by the theorem.
- (e) Compute $\lim_{x \rightarrow 3^-} f(x)$.
- (f) Compute $\lim_{x \rightarrow 3^+} f(x)$.
- (g) Compute $\lim_{x \rightarrow 3} f(x)$.

- (h) Compute $\lim_{x \rightarrow 5} |f(x)|$.
- (i) Suppose $A(x) = \int_0^x f(t) dt$, then compute $A(2)$.
- (j) Suppose $A(x)$ is defined as in the previous problem. What is $A''(0)$?
- (k) Suppose $h(x) = f(x^2)$, then compute $h'(-3)$.
- (l) Suppose $B(x) = \int_{-4}^x f(t) dt$. Compute $B(-2)$.
- (m) Let $B(x)$ be defined as in the previous problem. $B(x)$ has three critical points, what are they? Only one of these critical points is a local extrema, which one is it? Is it a local max or a local min?
- (n) Is the equation $f'(0) = 0$ true, false, or cannot be determined?
- (o) Is $f''(1.7) > 0$, $f''(1.7) < 0$, $f''(1.7) = 0$, or can it not be determined?
- (p) Compute $\lim_{x \rightarrow -2^-} \frac{f(x) - f(-2)}{x + 2}$.
- (q) Does $f(x)$ satisfy the Mean Value Theorem on $[-4, 0]$? If not, say which part of the theorem is not satisfied.
- (r) Over the interval $[3, 10]$, what are the absolute minimum and absolute maximum values of $f(x)$?
- (s) Compute $f(3)$.
- (t) Let $k(x) = -2f(x + 1) + 3$. Find $k(-5)$.
- (u) On the interval $(-10, -5)$, what would be an antiderivative of $f(x)$?
- (v) What is the domain of $f(x)$?
- (w) What is the domain of $f'(x)$?
- (x) Compute $\int_{-10}^{-6} -f(x) + 2 dx$.
- (y) For a function to be continuous at $x = a$, three conditions must be satisfied. Which of these three conditions fails for $f(x)$ at $x = -5$?
- (z) Compute $\lim_{x \rightarrow 8} \frac{f(x)}{e^{-x+8} - x + 7}$.