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Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet or on white unlined paper, and handed in at the end of class on the due date below. If you are working remotely, upload pictures of your written work to D2L by 11:59 PM on the due date below. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Use a PENCIL and if you make a mistake, use an eraser. Submitted work that does not demonstrate clearly the process by which one arrived at the answer will not receive credit of any kind. If a problem asks for an Excel or MATLAB solution, then upload your Excel/MATLAB program files to D2L as well. Academic dishonesty will not be tolerated.

Problem Set V

MAT 362-010 – Operations Research II

Due: Friday, May 5 by 11:59 PM on D2L

Read: Sections 14.1–14.3

Solutions!

Problem Number	Available Points	Your Points
1	8	8
2	8	8
Total	16	16

1. Graphically solve the following zero-sum game. The reward matrix is in terms of Player A. Once you found one player's strategy determine the strategy of the other player. [(8)]

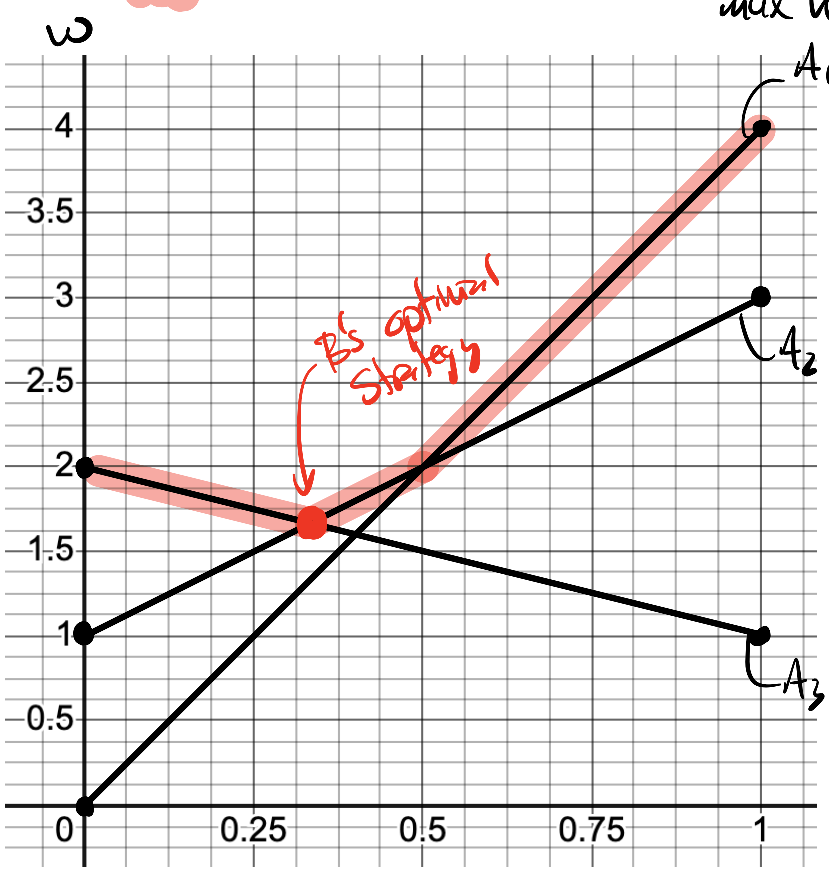
* Since Player B only has two strategies, we can solve B's game graphically.

	B_1	B_2	maximin = 1
A_1	4	0	0
A_2	3	1	1
A_3	1	2	1
	4	2	
minimax = 2			

$\boxed{1 < v = \omega < 2}$

A's Strategies	B's Expected losses
A_1	$4y_1 + 0y_2 \rightarrow \omega = 4y_1$
A_2	$3y_1 + y_2 = 3y_1 + 1 - y_1 \rightarrow \omega = 2y_1 + 1$
A_3	$y_1 + 2y_2 = y_1 + 2(1-y_1) \rightarrow \omega = -y_1 + 2$

– B's worst losses ● – B's minimum max losses



B's Solution:

Intersection of A_2 & A_3

$2y_1 + 1 = -y_1 + 2$

$3y_1 = 1$

$\boxed{y_1 = \frac{1}{3},\ y_2 = \frac{2}{3}}$

Value: $\boxed{\omega = 5/3}$

A's Solution

Since the value of the game is from A_2 & A_3, we know $x_1 = 0$.

$B_1:\quad 4x_1 + 3x_2 + x_3 = 3x_2 + x_3$

$B_2:\quad 0x_1 + x_2 + 2x_3 = x_2 + 2x_3$

$3x_2 + x_3 = x_2 + 2x_3$

$\Rightarrow 2x_2 = x_3$

$\Rightarrow 2x_2 = 1 - x_2$

$\Rightarrow \boxed{x_2 = \frac{1}{3},\ x_3 = \frac{2}{3}}$

Value: $\boxed{v = 5/3}$

2. Consider the following reward matrix for Player A for a zero-sum game. Formulate an LP for both players and solve both of them in Excel. Include your formulations and the solutions in the space below. Who wins the game? [(8)]

	B_1	B_2	B_3	B_4	B_5
A_1	1	−3	2	−2	1
A_2	2	3	0	3	−2
A_3	0	4	−1	−3	2
A_4	−4	0	−2	2	−1

Player A:

B's Strategy	A's Expectation
B_1	$x_1+2x_2+0x_3-4x_4$
B_2	$-3x_1+3x_2+4x_3+0x_4$
B_3	$2x_1+0x_2-x_3-2x_4$
B_4	$-2x_1+3x_2-3x_3+2x_4$
B_5	$x_1-2x_2+2x_3-x_4$

A's LP:

Maximize $Z=V$

Subject to

$V \le x_1+2x_2+0x_3-4x_4$

$V \le -3x_1+3x_2+4x_3+0x_4$

$V \le 2x_1+0x_2-x_3-2x_4$

$V \le -2x_1+3x_2-3x_3+2x_4$

$V \le x_1-2x_2+2x_3-x_4$

$x_1+x_2+x_3+x_4=1$

$V = \text{urs}, \ x_i \ge 0$

Player B:

A's Strategy	B's Expectation
A_1	$y_1-3y_2+2y_3-2y_4+y_5$
A_2	$2y_1+3y_2+0y_3+3y_4-2y_5$
A_3	$0y_1+4y_2-y_3-3y_4+2y_5$
A_4	$-4y_1+0y_2-2y_3+2y_4-y_5$

B's LP:

Minimize $Z=\omega$

Subject to

$\omega \ge y_1-3y_2+2y_3-2y_4+y_5$

$\omega \ge 2y_1+3y_2+0y_3+3y_4-2y_5$

$\omega \ge 0y_1+4y_2-y_3-3y_4+2y_5$

$\omega \ge -4y_1+0y_2-2y_3+2y_4-y_5$

$y_1+y_2+y_3+y_4+y_5=1$

$\omega = \text{urs}, \ y_i \ge 0$

Excel Solution:

$x_1 = .31$ $x_3 = .21$

$x_2 = .27$ $x_4 = .22$

$V = -.01899 = \omega$

Excel Solution:

$y_1 = .013$ $y_3 = .063$ $y_5 = .563$

$y_2 = 0$ $y_4 = .361$

PLAYER B WINS!