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**Instructions:** All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Desmos graphs can be submitted separately.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. This assignment is graded on effort, completeness, and neatness for a total of 5 points. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer may result in a loss of points. Any parts to any questions that are not answered will also result in a loss of points. Academic dishonesty will not be tolerated.

# Problem Set V

## MAT 181 – Calculus I

Due: Friday, March 29, by 11:59 PM on D2L

Read: Sections 3.5–3.7

Solutions!

1. Differentiate the following relations implicitly to find $\frac{dy}{dx}$:

(a) $3x^4 + 2y^3 = 10834$

$$\frac{d}{dx}(3x^4) + \frac{d}{dx}(2y^3) = \frac{d}{dx}(10834)$$

$$\Rightarrow \quad 12x^3 + 6y^2\frac{dy}{dx} = 0 \quad \Rightarrow \quad 6y^2\frac{dy}{dx} = -12x^3 \quad \Rightarrow \quad \boxed{\frac{dy}{dx} = -\frac{2x^3}{y^2}}$$

(b) $15x^2 - 4y^6 = 2x^2y^3$

$$\frac{d}{dx}(15x^2) - \frac{d}{dx}(4y^6) = \frac{d}{dx}(2x^2y^3)$$

$$\Rightarrow \quad 30x - 24y^5\frac{dy}{dx} = 4xy^3 + 2x^2 \cdot 3y^2\frac{dy}{dx}$$

$$\Rightarrow \quad 30x - 4xy^3 = 24y^5\frac{dy}{dx} + 6x^2y^2\frac{dy}{dx}$$

$$\Rightarrow \quad 30x - 4xy^3 = (24y^5 + 6x^2y^2)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{30x - 4xy^3}{24y^5 + 6x^2y^2}}$$

(c) $\cos(x+y) = y^2e^x + 69$

$$\frac{d}{dx}(\cos(x+y)) = \frac{d}{dx}(y^2e^x) + \frac{d}{dx}(69)$$

$$\Rightarrow \quad -\sin(x+y)\cdot\frac{d}{dx}(x+y) = 2y\frac{dy}{dx}e^x + y^2e^x$$

$$\Rightarrow \quad -\sin(x+y)\cdot\left(1 + \frac{dy}{dx}\right) = 2ye^x\frac{dy}{dx} + y^2e^x$$

$$\Rightarrow \quad -\sin(x+y) - \sin(x+y)\frac{dy}{dx} = 2ye^x\frac{dy}{dx} + y^2e^x$$

$$\Rightarrow \quad -\sin(x+y)\frac{dy}{dx} - 2ye^x\frac{dy}{dx} = \sin(x+y) + y^2e^x$$

$$\Rightarrow \quad (-\sin(x+y) - 2ye^x)\frac{dy}{dx} = \sin(x+y) + y^2e^x$$

$$\boxed{\frac{dy}{dx} = \frac{\sin(x+y) + y^2e^x}{-\sin(x+y) - 2ye^x}}$$

2. Consider the curve defined by the equation $x^3 + y^2 = \frac{5}{8}xy^3$. Verify that $(1, 2)$ lies on this curve. Find the equation of the tangent line to the curve at the point $(1, 2)$. Submit a Desmos graph of the curve defined by the equation and the tangent line on the same plot.

<u>Verify</u>: $1^3 + 2^2 \stackrel{?}{=} \frac{5}{8}(1)(2)^3 \Rightarrow 1+4 \stackrel{?}{=} \frac{5}{8}\cdot 8 \Rightarrow 5 = 5$

The point $(1,2)$ is on the curve.

<u>Slope</u>: $\frac{d}{dx}(x^3) + \frac{d}{dx}(y^2) = \frac{d}{dx}\left(\frac{5}{8}xy^3\right)$

$$\Rightarrow 3x^2 + 2y\frac{dy}{dx} = \frac{5}{8}y^3 + \frac{5}{8}x\cdot 3y^2\frac{dy}{dx}$$

$$\Rightarrow 2y\frac{dy}{dx} - \frac{15}{8}xy^2\frac{dy}{dx} = \frac{5}{8}y^3 - 3x^2$$

$$\Rightarrow \left(2y - \frac{15}{8}xy^2\right)\frac{dy}{dx} = \frac{5}{8}y^3 - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{5}{8}y^3 - 3x^2}{2y - \frac{15}{8}xy^2} \Rightarrow \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{\frac{5}{8}(2)^3 - 3(1)^2}{2(2) - \frac{15}{8}(1)(2)^2} = \frac{5-3}{4-\frac{15}{2}} = -\frac{4}{7}$$

<u>Tangent Line</u>: $y = mx + b$

$$2 = -\frac{4}{7}(1) + b$$

$$\frac{14}{7} + \frac{4}{7} = b$$

$$\frac{18}{7} = b$$

$$\boxed{y = -\frac{4}{7}x + \frac{18}{7}}$$

3. Find the equation of the line tangent to the graph of $f(x) = \dfrac{(4+x)2^{\ln(x)}}{x^3 2^x}$ at the point $(1, f(1))$. **Do not use any decimal approximations.**

Slope: $f'(x) = \dfrac{[(4+x)2^{\ln(x)}]'(x^3 2^x) - (4+x)2^{\ln(x)}[x^3 2^x]'}{(x^3 2^x)^2}$

$\Rightarrow f'(x) = \dfrac{[2^{\ln(x)} + (4+x)\ln(2)2^{\ln(x)}\cdot\frac{1}{x}](x^3 2^x) - (4+x)2^{\ln(x)}[3x^2 2^x + x^3\ln(2)\cdot 2^x]}{(x^3 2^x)^2}$

$\Rightarrow f'(1) = \dfrac{[2^{\ln(1)} + (4+1)\ln(2)2^{\ln(1)}\cdot\frac{1}{1}](1^3 2^1) - (4+1)2^{\ln(1)}[3(1)^2 2^1 + (1)^3\ln(2)\cdot 2^1]}{(1^3\cdot 2^1)^2}$

$\Rightarrow f'(1) = \dfrac{[2^0 + 5\ln(2)\cdot 2^0](2) - 5\cdot 2^0[6 + 2\ln(2)]}{4}$

$\Rightarrow f'(1) = \dfrac{[1 + 5\ln(2)]\cdot 2 - 5[6 + 2\ln(2)]}{4}$

$\Rightarrow f'(1) = \dfrac{2 + \cancel{10\ln(2)} - 30 - \cancel{10\ln(2)}}{4}$

$\Rightarrow f'(1) = \dfrac{-28}{4}$

$\Rightarrow f'(1) = -7$

$f(1) = \dfrac{(4+1)2^{\ln(1)}}{1^3\cdot 2^1} = \dfrac{5}{2}$

Tangent Line:

point: $(1, f(1)) = (1, 5/2)$

slope: $m = f'(1) = -7$

$\Rightarrow y = mx + b$

$\frac{5}{2} = -7(1) + b$

$b = \frac{5}{2} + \frac{14}{2} = \frac{19}{2}$

$\boxed{y = -7x + \frac{19}{2}}$

4. For each of the following functions, find the derivative function $f'(x)$ using derivative rules developed in class. Simplify as much as humanly possible.

   (a) $f(x) = \arcsin(2\sqrt{x})$

$$f'(x) = \frac{1}{\sqrt{1-(2\sqrt{x})^2}} \cdot \frac{2}{2\sqrt{x}}$$

$$\Rightarrow \quad f'(x) = \frac{1}{\sqrt{x}\sqrt{1-4x}}$$

   (b) $f(x) = \ln(x^{.18} + 4x - 6)$

$$f'(x) = \frac{1}{x^{.18}+4x-6} \cdot (.18x^{-.82}+4)$$

$$\Rightarrow \quad f'(x) = \frac{.18x^{-.82}+4}{x^{.18}+4x-6}$$

   (c) $f(x) = 2^{\arctan(x+2)}$

$$f'(x) = \ln(2) \cdot 2^{\arctan(x+2)} \cdot \frac{1}{1+(x+2)^2} \cdot 1$$

$$\Rightarrow \quad f'(x) = \frac{\ln(2) \cdot 2^{\arctan(x+2)}}{1+(x+2)^2}$$

   (d) $f(x) = \log_7(\log_5(\log_2(x)))$

$$f'(x) = \frac{1}{\ln(7) \cdot \log_5(\log_2(x))} \cdot \frac{1}{\ln(5)\log_2(x)} \cdot \frac{1}{\ln(2) \cdot x}$$

$$\Rightarrow \quad f'(x) = \frac{1}{\ln(7)\ln(5)\ln(2)\,x\,\log_2(x)\,\log_5(\log_2(x))}$$

5. Application Problem: A boat is towed toward a dock by a cable attached to a winch that stands 10 feet above the water level (see figure below). Let $\theta$ be the angle of elevation of the winch and let $\ell$ be the length of the cable as the boat is towed toward the dock.

   (a) Find $\dfrac{d\theta}{d\ell}$ in terms of $\ell$ only.

From the figure, we have

$$\sin(\theta) = \frac{10}{\ell}$$

$$\Rightarrow \quad \theta = \arcsin\left(\frac{10}{\ell}\right)$$

$$\Rightarrow \quad \frac{d\theta}{d\ell} = \frac{1}{\sqrt{1-\left(\frac{10}{\ell}\right)^2}} \cdot \left(-\frac{10}{\ell^2}\right)$$

$$\frac{d\theta}{d\ell} = \frac{-10}{\ell^2\sqrt{1-\frac{100}{\ell^2}}}$$

$$\Rightarrow \boxed{\frac{d\theta}{d\ell} = \frac{-10}{\ell\sqrt{\ell^2-100}}}$$

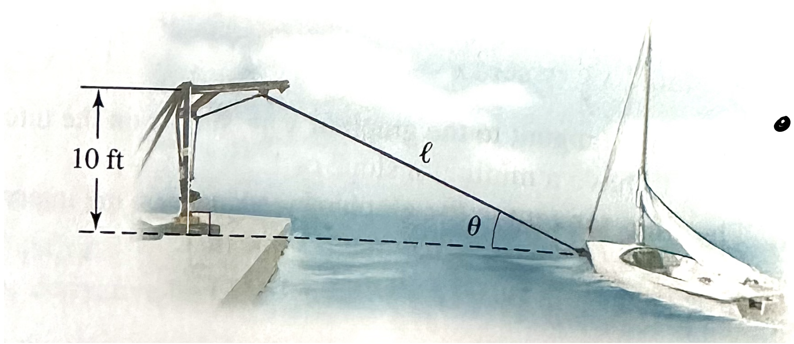
   (b) Compute $\dfrac{d\theta}{d\ell}$ when $\ell = 50$, 20, and 11 ft. Explain what would happen as $\ell \to 10^+$. Notice from the figure that $\theta$ is increasing as the boat is getting closer to the dock. Why, then, is $d\theta/d\ell$ negative?

$$\left.\frac{d\theta}{d\ell}\right|_{\ell=50} = \frac{-10}{50\sqrt{50^2-100}} = -0.00408 \text{ radians/foot}$$

$$\left.\frac{d\theta}{d\ell}\right|_{\ell=20} = \frac{-10}{20\sqrt{20^2-100}} = -0.02887 \text{ radians/foot}$$

$$\left.\frac{d\theta}{d\ell}\right|_{\ell=11} = \frac{-10}{11\sqrt{11^2-100}} = -0.19838 \text{ radians/foot}$$

- As $\ell \to 10^+$, $\frac{d\theta}{d\ell} \to -\infty$.




- $\frac{d\theta}{d\ell}$ is negative because although the change in angle is positive ($\theta$ is increasing), the change in length is negative ($\ell$ is decreasing).
- Hence, $d\theta > 0$, $d\ell < 0 \Rightarrow \frac{d\theta}{d\ell} < 0$.