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Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Submit any Excel or Python files as well.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer will not receive credit of any kind. Academic dishonesty will not be tolerated.

Problem Set IV

MAT 362-010 – Operations Research II

Due: Friday, April 19 by 11:59 PM on D2L

Read: Sections 14.1–14.3

Solutions!

Problem Number	Available Points	Your Points
1	8	8
2	4	4
3	8	8
Total	20	20

1. Graphically solve the following zero-sum game. The reward matrix is in terms of Player A. Once you found one player's strategy determine the strategy of the other player. [(8)]

* Since Player B only has two strategies, we can solve B's game graphically.

	B_1	B_2	
A_1	4	0	0
A_2	3	1	1
A_3	1	2	1
	4	2	

maximin = 1

minimax = 2

$\boxed{1 < v = \omega < 2}$

A's Strategies	B's Expected losses
A_1	$4y_1 + 0y_2 \rightarrow \omega = 4y_1$
A_2	$3y_1 + y_2 = 3y_1 + 1 - y_1 \rightarrow \omega = 2y_1 + 1$
A_3	$y_1 + 2y_2 = y_1 + 2(1-y_1) \rightarrow \omega = -y_1 + 2$

B's Solution:

Intersection of A_2 & A_3

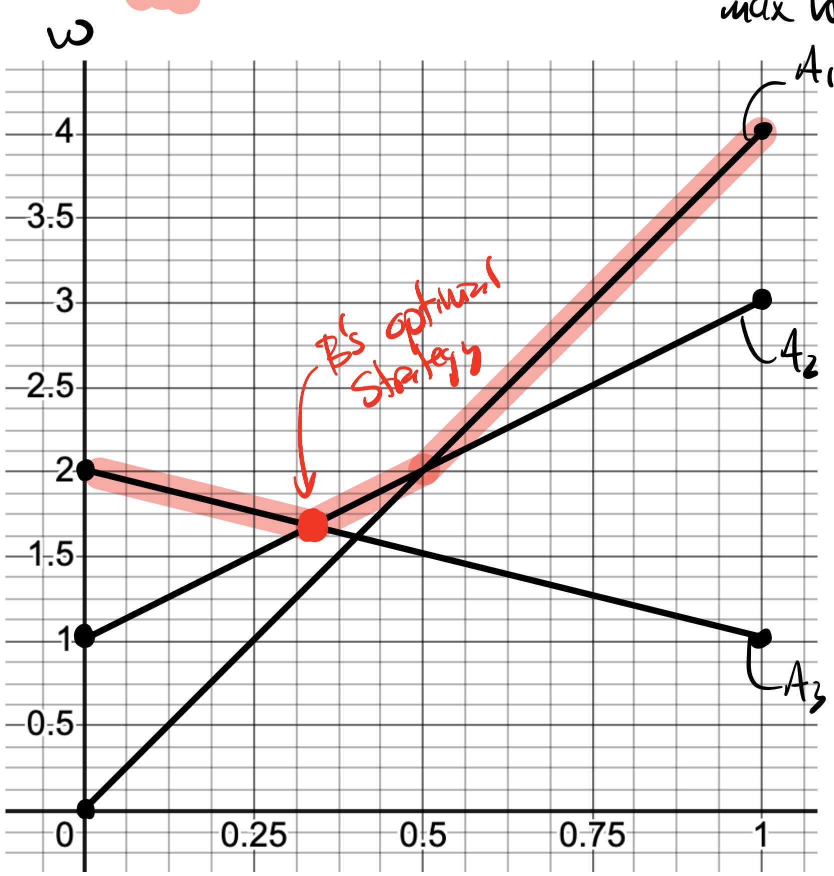
$2y_1 + 1 = -y_1 + 2$

$3y_1 = 1$

$\boxed{y_1 = \frac{1}{3},\ y_2 = \frac{2}{3}}$

Value: $\boxed{\omega = 5/3}$

– B's worst losses   – B's minimum max losses



A's Solution

Since the value of the game is from A_2 & A_3, we know $X_1 = 0$.

B_1: $4X_1 + 3X_2 + X_3 = 3X_2 + X_3$

B_2: $0X_1 + X_2 + 2X_3 = X_2 + 2X_3$

$3X_2 + X_3 = X_2 + 2X_3$

$\Rightarrow 2X_2 = X_3$

$\Rightarrow 2X_2 = 1 - X_2$

$\Rightarrow \boxed{X_2 = \frac{1}{3},\ X_3 = \frac{2}{3}}$

Value: $\boxed{v = 5/3}$

2. Two companies promote two competing products. Currently, each product controls 50% of the market. Because of recent improvements in the two products, each company plans to launch an advertising campaign. If neither company advertises, equal market shares will continue. If either company launches a stronger campaign, the other company is certain to lose a proportional percentage of its customers. A survey of the market shows that 50% of potential customers can be reached through television, 30% through newspapers, and 20% through radio. [(4)]

   (a) Formulate the problem as a two-person zero-sum game, and determine the advertising media for each company.

   (b) Determine a range for the value of the game. Can each company operate with a single pure strategy?

a.) Define the following strategies for each company:

1- no campaign
2- TV
3- newspaper
4- radio
5- TV and newspaper
6- TV and radio
7- radio and newspaper
8- TV, radio, and newspaper

Let the following payout matrix be for Company A. Each entry represents the percentage of the other company's market share won/lost.

B's Strategies

A's Strategies	1	2	3	4	5	6	7	8	
1	0	-50	-30	-20	-80	-70	-50	-100	-100
2	50	0	20	30	-30	-20	0	-50	-50
3	80	-20	0	10	-50	-40	-20	-70	-70
4	20	-30	-10	0	-60	-50	-30	-80	-80
5	80	30	50	60	0	10	30	-20	-20
6	70	20	40	50	-10	0	20	-30	-30
7	50	0	20	30	-30	-20	0	-50	-50
8	100	50	70	80	20	30	50	0	0
	100	50	70	80	20	30	50	$\boxed{0}$	

$v=0$

Pure Strategy for each team: Do all forms of advertisement!

Game is fair.

3. Consider the following reward matrix for Player A for a zero-sum game. Formulate an LP for both players and solve both of them in Excel. Include your formulations and the solutions in the space below. Who wins the game? [(8)]

	B_1	B_2	B_3	B_4	B_5
A_1	1	−3	2	−2	1
A_2	2	3	0	3	−2
A_3	0	4	−1	−3	2
A_4	−4	0	−2	2	−1

Player A:

B's Strategy	A's Expectation
B_1	$x_1+2x_2+0x_3-4x_4$
B_2	$-3x_1+3x_2+4x_3+0x_4$
B_3	$2x_1+0x_2-x_3-2x_4$
B_4	$-2x_1+3x_2-3x_3+2x_4$
B_5	$x_1-2x_2+2x_3-x_4$

A's LP:

Maximize $Z=V$

Subject to

$V \le x_1+2x_2+0x_3-4x_4$

$V \le -3x_1+3x_2+4x_3+0x_4$

$V \le 2x_1+0x_2-x_3-2x_4$

$V \le -2x_1+3x_2-3x_3+2x_4$

$V \le x_1-2x_2+2x_3-x_4$

$x_1+x_2+x_3+x_4=1$

$V = \text{urs},\ x_i \ge 0$

Player B:

A's Strategy	B's Expectation
A_1	$y_1-3y_2+2y_3-2y_4+y_5$
A_2	$2y_1+3y_2+0y_3+3y_4-2y_5$
A_3	$0y_1+4y_2-y_3-3y_4+2y_5$
A_4	$-4y_1+0y_2-2y_3+2y_4-y_5$

B's LP:

Minimize $Z=\omega$

Subject to

$\omega \ge y_1-3y_2+2y_3-2y_4+y_5$

$\omega \ge 2y_1+3y_2+0y_3+3y_4-2y_5$

$\omega \ge 0y_1+4y_2-y_3-3y_4+2y_5$

$\omega \ge -4y_1+0y_2-2y_3+2y_4-y_5$

$y_1+y_2+y_3+y_4+y_5=1$

$\omega = \text{urs},\ y_i \ge 0$

Excel Solution:

$x_1 = .31$ $x_3 = .21$

$x_2 = .27$ $x_4 = .22$

$V = -.01899 = \omega$

Excel Solution:

$y_1 = .013$ $y_3 = .063$ $y_5 = .563$

$y_2 = 0$ $y_4 = .361$

PLAYER B WINS!