Name: ____________________

Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Desmos graphs can be submitted separately.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. This assignment is graded on effort, completeness, and neatness for a total of 5 points. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer may result in a loss of points. Any parts to any questions that are not answered will also result in a loss of points. Academic dishonesty will not be tolerated.

Problem Set III

MAT 181 – Calculus I

Due: Friday, February 23 by 11:59 PM on D2L

Read: Sections 2.1, 2.5, 2.6, and 3.1

1. Consider the following piecewise function below. Determine if the function is continuous at $x = 0$ and $x = 4$ by computing, explicitly, all necessary limits. (Note: you are not allowed to compute limits graphically.) If it is not continuous at a point, determine if it is left- or right-continuous or neither. Show all work to receive full credit.

$$f(x) = \begin{cases} \dfrac{\sin(x)}{x} & \text{if } x < 0 \\ \dfrac{x^3 - x^2 - 12x}{x^2 - 3x - 4} & \text{if } 0 \le x < 4 \\ \sqrt{x} + \dfrac{18}{5} & \text{if } x \ge 4 \end{cases}$$

2. Suppose that a sailboat is observed, over a period of 5 minutes, to travel a distance from a starting point according to the function $s(t) = t^3 + 60t$, where t is time in minutes and s is the distance traveled in meters.

(a) What is the average velocity of the boat between 1 and 3 minutes?

(b) What is the average velocity of the boat between 3 and 5 minutes?

(c) Fill out the tables below and be sure not to round too much.

Time interval	$[3, 3.5]$	$[3, 3.1]$	$[3, 3.01]$	$[3, 3.001]$
Change in time (Δt)				
Change in distance (Δs)				
Average velocity ($\frac{\Delta s}{\Delta t}$)				

Time interval	$[2.5, 3]$	$[2.9, 3]$	$[2.99, 3]$	$[2.999, 3]$
Change in time (Δt)				
Change in distance (Δs)				
Average velocity ($\frac{\Delta s}{\Delta t}$)				

(d) Using your tables above, make a conjecture about the instantaneous velocity of the boat at 3 minutes into its trip.

3. (This problem spans two pages.) The following questions pertain to the limit definition of the derivative either at a single point c or the derivative function. In each question below, **you must use the specified limit definition to compute the derivative to receive full credit.**

   (a) Using the limit definition of the derivative at a point c, namely $f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$, compute the derivative of $f(x) = 2x^2 + x$ at $c = 1$.

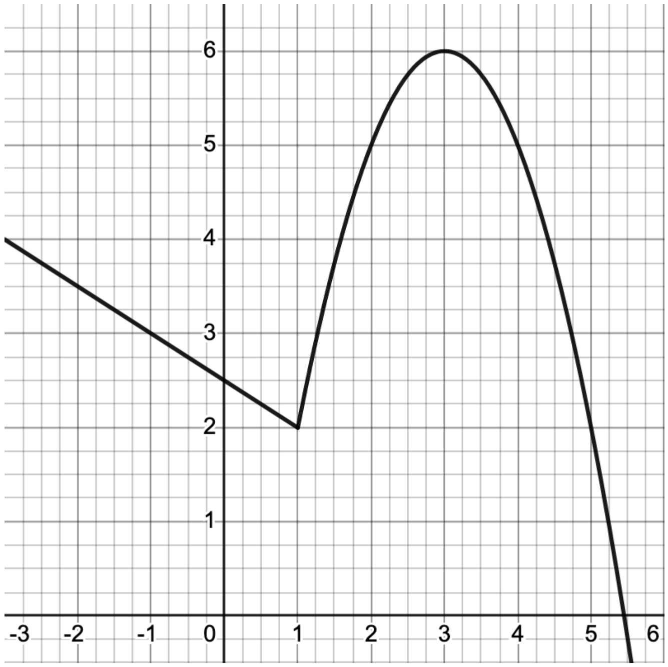
   (b) Using the limit definition of the derivative at a point c, namely $f'(c) = \lim\limits_{h \to 0} \dfrac{f(c+h) - f(c)}{h}$, compute the derivative of $f(x) = \sqrt{2x}$ at $c = 2$.

(c) Using the limit definition of the derivative function, namely $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, compute the derivative function of the functions below. Specify, in interval notation, where f is continuous and where f is differentiable.

i. $f(x) = \sqrt{3 - x}$

ii. $f(x) = -\frac{5}{x^2}$

4. Consider the piecewise function $f(x)$ given below with its graph. Answer parts $(a) - (d)$ simply by looking at the graph.



$$f(x) = \begin{cases} -\frac{1}{2}x + \frac{5}{2} & \text{if } x \leq 1 \\ -x^2 + 6x - 3 & \text{if } x > 1 \end{cases}$$

(a) $f(-1) =$

(b) $f'(-1) =$

(c) $f(3) =$

(d) $f'(3) =$

(e) Notice from the graph that $f(x)$ is continuous at $x = 1$. Show that $f(x)$ is not differentiable $x = 1$ by **explicitly computing the left- and right-side limits of the derivative**.

5. Application Problem: The owner of a small toy manufacturer has determined that he can sell x toys if the price is $D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars. (Hint: to do this problem, you might have to *do some light research* about price, cost, and profit functions.)

   (a) Find any break-even points. (Hint: you have to find the *revenue* function first.)

   (b) Find the profit function, $P(x)$, and submit a Desmos graph of this function. What do the zeros of this function represent?

   (c) Compute the marginal profit function using the limit definition of the derivative.