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Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Desmos graphs can be submitted separately.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. This assignment is graded on effort, completeness, and neatness for a total of 5 points. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer may result in a loss of points. Any parts to any questions that are not answered will also result in a loss of points. Academic dishonesty will not be tolerated.

Problem Set II

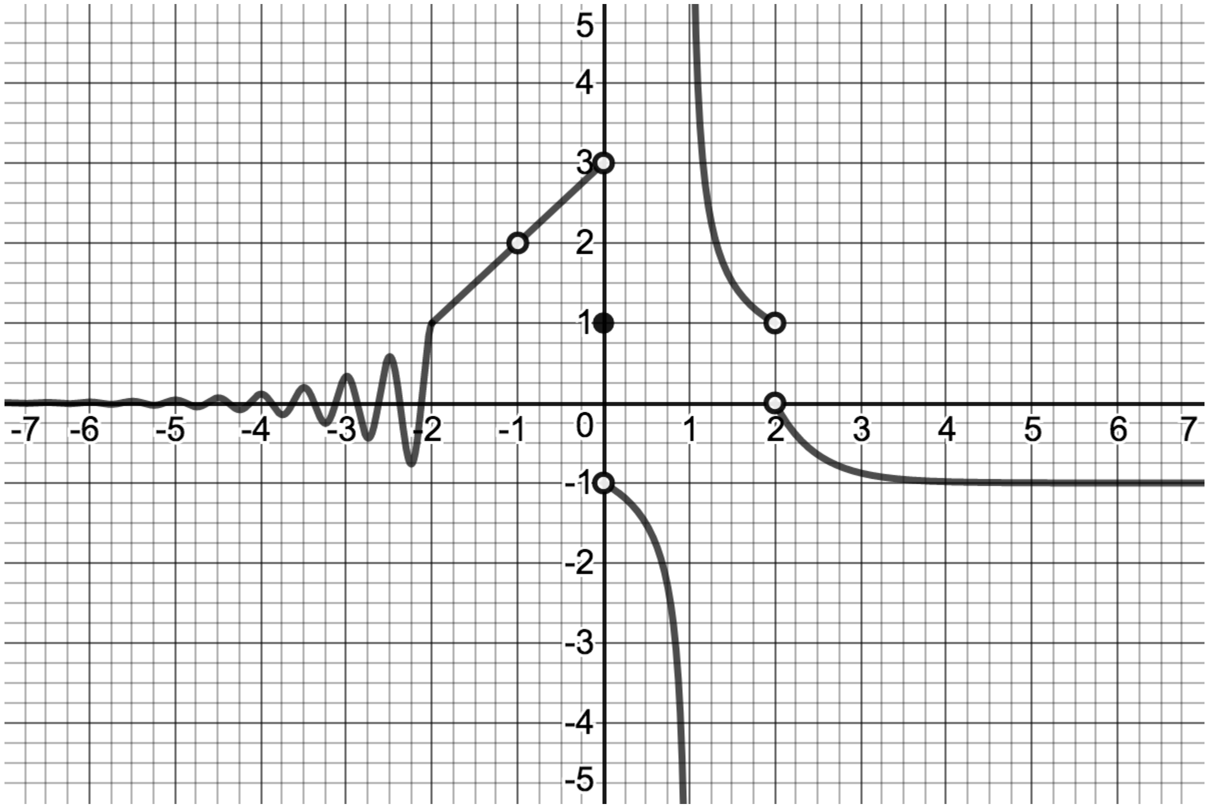
MAT 181-050 – Calculus I

Due: Friday, February 9 by 11:59 PM on D2L

Read: Sections 1.4, 2.2, and 2.4

Solutions!

1. Consider the graph of the function $f(x)$ below:



(a) Use the graph to fill in the entries of the table below. If the limit exists, write the value of the limit. If the limit is infinite, write ∞ or $-\infty$. If the limit does not exist, write DNE.

	$c=-2$	$c=-1$	$c=0$	$c=1$	$c=2$
$\lim_{x\to c^-} f(x)$	1	2	3	$-\infty$	1
$\lim_{x\to c^+} f(x)$	1	2	-1	∞	0
$\lim_{x\to c} f(x)$	1	2	DNE	DNE	DNE
$f(c)$	1	DNE	1	DNE	DNE

(b) Does this function have any horizontal or vertical asymptotes? **If so, write the appropriate limit(s) to justify your answer.**

Horizontal Asymptotes: $\lim_{x\to\infty} f(x) = -1 \to \boxed{y=-1}$

$\lim_{x\to-\infty} f(x) = 0 \to \boxed{y=0}$

Vertical Asymptote: $\left.\begin{array}{l}\lim_{x\to1^-} f(x) = -\infty \\ \lim_{x\to1^+} f(x) = \infty\end{array}\right\} \to \boxed{x=1}$

2. Let $f(x) = \dfrac{e^x - x^2 - 1}{x}$.

(a) Is $f(x)$ algebraic or transcendental?

transcendental, the function contains e^x

(b) What is the domain of $f(x)$?

Domain: $(-\infty, 0) \cup (0, \infty)$ (any real # but 0)

(c) What is $f(0)$?

f(0) = DNE because 0 is not in the domain of f.

(d) Evaluate this limit $\lim_{x\to 0} f(x)$ using the "Brute Force" method. Define this function in Desmos and evaluate the function at the appropriate points to fill in the table below, then make a conclusion about the limit in the space below the table. Round all of your answer to **six decimal places**.

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	1.051626	1.005017	1.0005002	1.00005000

$$\lim_{x\to 0^-} \frac{e^x - x^2 - 1}{x} = 1$$

x	0.1	0.01	0.001	0.0001
$f(x)$	.951709	.995017	.9995002	.99995000

$$\lim_{x\to 0^+} \frac{e^x - x^2 - 1}{x} = 1$$

Using the evidence gathered above, deduce the value of the following limit, if it exists:

$$\lim_{x\to 0} \frac{e^x - x^2 - 1}{x} = 1$$

(e) Graph the function in Desmos. Submit a figure of the graph on D2L that supports your answer to part *(d)*.

(graph is attached)

3. For parts (a)–(c) below, algebraically determine the limits. You must show *at least* two intermediate steps to receive full credit. For parts (d)–(f) below, use the brute force method or provide a justification using complete sentences to determine the limits.

(a) $\lim_{x\to -1} \frac{x^2-2x-3}{x+1}$ Rational function, domain is $(-\infty,-1)\cup(-1,\infty)$

$= \lim_{x\to -1} \frac{\cancel{(x+1)}(x-3)}{\cancel{x+1}} = \lim_{x\to -1} x-3 = -1-3 = \boxed{-4}$

There is a hole in the graph at $(-1,-4)$.

(b) $\lim_{x\to 6} \frac{x-6}{\sqrt{x}-\sqrt{6}}$ Algebraic function, domain $[0,6)\cup(6,\infty)$

$= \lim_{x\to 6} \frac{(x-6)(\sqrt{x}+\sqrt{6})}{(\sqrt{x}-\sqrt{6})(\sqrt{x}+\sqrt{6})} = \lim_{x\to 6} \frac{\cancel{(x-6)}(\sqrt{x}+\sqrt{6})}{\cancel{x-6}} = \lim_{x\to 6} \sqrt{x}+\sqrt{6} = \sqrt{6}+\sqrt{6} = \boxed{2\sqrt{6}}$

There is a hole in the graph at $(6, 2\sqrt{6})$

(c) $\lim_{x\to 4} \frac{x-1}{\sqrt{4x}-2}$ Algebraic Function, Domain: $[0,1)\cup(1,\infty)$

$= \lim_{x\to 4} \frac{x-1}{\sqrt{4x}-2} = \frac{4-1}{\sqrt{4\cdot 4}-2} = \frac{3}{\sqrt{16}-2} = \frac{3}{4-2} = \boxed{\frac{3}{2}}$

The graph goes right through the point $(4, 3/2)$.

(d) $\lim_{x\to\infty} \frac{3^x}{2^x}$ Domain: $(-\infty,\infty)$

$= \lim_{x\to\infty} \left(\frac{3}{2}\right)^x \leftarrow$ Exponential function that grows without bound $= \boxed{\infty}$

(e) $\lim_{x\to\infty} x^2-2x-9$ Domain: $(-\infty,\infty)$

$= \lim_{x\to\infty} x^2\left(1-\frac{2x}{x^2}-\frac{9}{x^2}\right) = \left(\lim_{x\to\infty} x^2\right)\left(\lim_{x\to\infty} 1-\frac{2}{x}-\frac{9}{x^2}\right) = (\infty)(1) = \boxed{\infty}$

(f) $\lim_{x\to 10^-} \frac{5}{x-10}$ Domain: $(-\infty,10)\cup(10,\infty)$

$x=9.99: \frac{5}{9.99-10} = \frac{5}{-.01} = -500$

$x=9.999: \frac{5}{9.999-10} = \frac{5}{-.001} = -5000 \longrightarrow \lim_{x\to 10^-} \frac{5}{x-10} = \boxed{-\infty}$

(g) $\lim_{x\to \frac{\pi}{4}^-} \sec(x)$ Domain: $x \neq \frac{\pi}{2}+n\pi$, n is an integer.

Note: $\frac{\pi}{4}$ is in the domain.

$= \lim_{x\to \pi/4^-} \sec(x) = \sec(\pi/4) = \frac{1}{\cos(\pi/4)} = \frac{1}{1/\sqrt{2}} = \boxed{\sqrt{2}}$

4. Algebraically evaluate the following limits. You must show a sequence of steps to receive full credit.

(a) $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$, where x is any non-negative constant.

Cannot plug in $h=0$!

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

(now plug in $h=0$!)

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

(b) $\lim_{x\to -3} \frac{\frac{1}{3}+\frac{1}{x}}{x^2-9}$

Algebraic function, Domain is $(-\infty,-3)\cup(-3,0)\cup(0,3)\cup(3,\infty)$

$$= \lim_{x\to -3} \frac{1}{x^2-9}\left(\frac{1}{3}+\frac{1}{x}\right)$$

$$= \lim_{x\to -3} \frac{1}{(x+3)(x-3)}\left(\frac{x}{3x}+\frac{3}{3x}\right)$$

$$= \lim_{x\to -3} \frac{1}{\cancel{(x+3)}(x-3)}\left(\frac{\cancel{x+3}}{3x}\right)$$

$$= \lim_{x\to -3} \frac{1}{x-3}\left(\frac{1}{3x}\right)$$

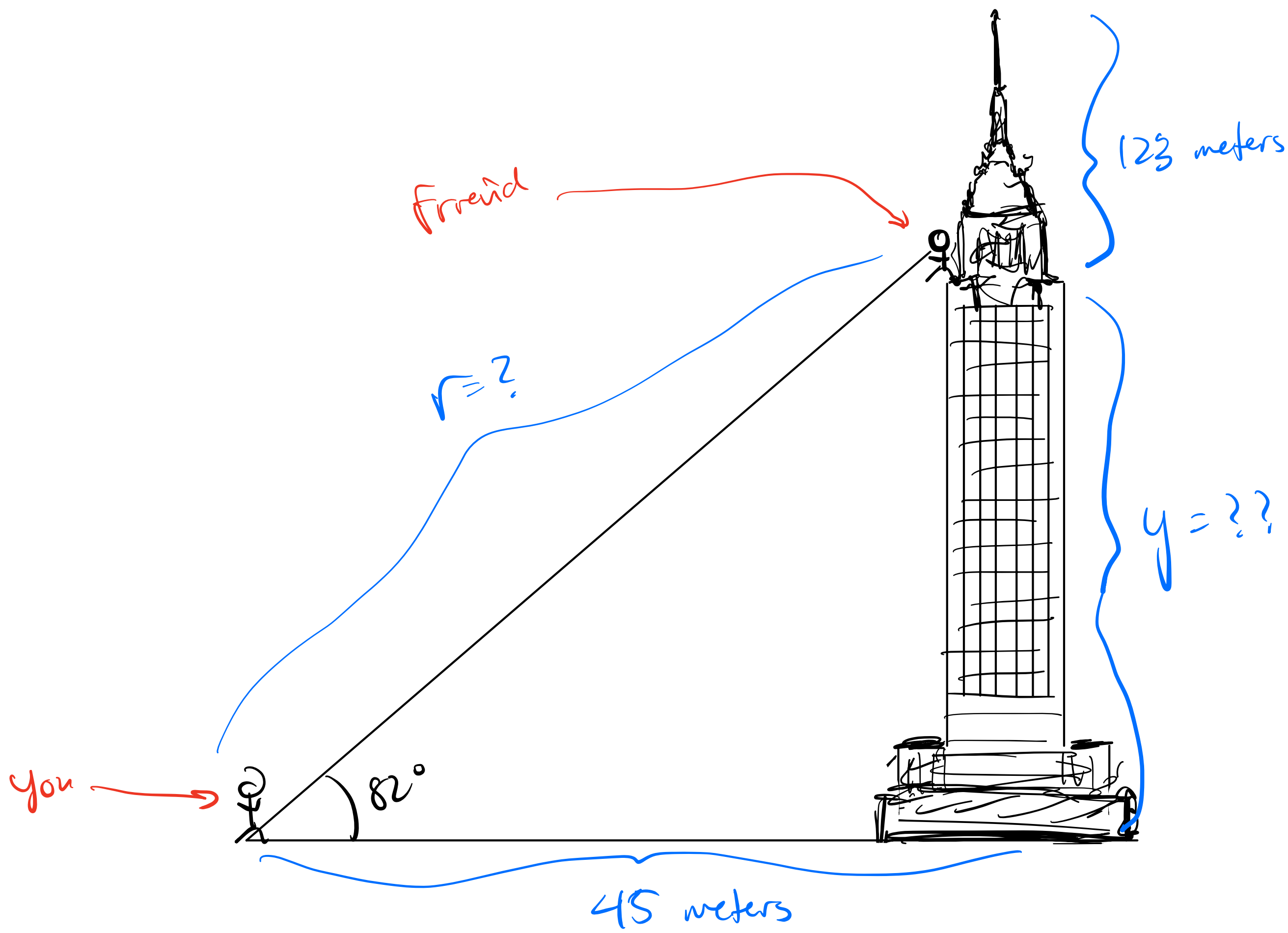
$$= \frac{1}{-3-3}\cdot\frac{1}{3(-3)}$$

$$= \frac{1}{-6}\cdot\frac{1}{-9}$$

$$= \boxed{\frac{1}{54}}$$

There is a hole in the graph at $(-3, \frac{1}{54})$.

5. Application Problem: You are standing 45 meters from the base of the Empire State Building. You estimate that the angle of elevation to the top of the 86th floor (the observatory) is 82°. If the total height of the building is another 123 meters above the 86th floor, what is the approximate height of the building? One of your friends is on the 86th floor. What is the distance between you and your friend?



Finding y:

$\tan(82°) = \frac{y}{45}$

$\Rightarrow y = 45 \cdot \tan(82°)$

$y = 320.19$ meters

[Height of Building] $\approx y + 123 = \boxed{443.19 \text{ m}}$

Finding r:

$\cos(82°) = \frac{45}{r}$

$\Rightarrow r\cos(82°) = 45$

$\Rightarrow r = \frac{45}{\cos(82°)}$

$\Rightarrow \boxed{r = 323.34 \text{ m}}$