Name: _____

> **Instructions:** All solutions should be prepared carefully and neatly. All solution sets shall
> be completed on this packet and submitted by uploading a scan or picture of your written
> work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your**
> **entire packet. Submit any Excel or Python files as well.** The mobile app called
> *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. Careless
> presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in
> a deduction of points at my discretion. Submitted work that does not demonstrate clearly
> the process by which one arrived at the answer will not receive credit of any kind. Academic
> dishonesty will not be tolerated.

Problem Set II

MAT 362-010 – Operations Research II

Due: Friday, March 1 by 11:59 PM on D2L

Read: Sections 6.7–6.10, 7.1–7.3

Problem Number	Available Points	Your Points
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
Total	40	

1. Consider the following LP: [(5)]

$$\text{Maximize:} \quad z = 5x_1 + 2x_2 + 3x_3$$

$$\text{Subject to:} \quad x_1 + 5x_2 + 2x_3 = 30$$
$$x_1 - 5x_2 - 6x_3 \leq 40$$
$$x_1, x_2, x_3 \geq 0$$

Suppose the optimal tableau for this problem is below and that $M = 100$ when solving the problem.

Row	Basic	z	x_1	x_2	x_3	a_1	s_2	RHS
0	z	1	0	23	7	105	0	150
1	x_1	0	1	5	2	1	0	30
2	s_2	0	0	-10	-8	-1	1	10

Write the associated dual problem and determine its optimal solution in two different ways by observation above and by computing $\boldsymbol{c}_{BV} B^{-1}$.

2. Consider the following LP: [(5)]

$$\text{Minimize:} \quad z = 5x_1 + 2x_2$$

$$\text{Subject to:} \quad x_1 - x_2 \geq 3$$
$$2x_1 + 3x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Determine whether or not the following pairs of primal-dual solutions are optimal:

a.) Primal: $x_1 = 3$, $x_2 = 1$. Dual: $y_1 = 4$, $y_2 = 1$

b.) Primal: $x_1 = 4$, $x_2 = 1$. Dual: $y_1 = 1$, $y_2 = 0$

c.) Primal: $x_1 = 3$, $x_2 = 0$. Dual: $y_1 = 5$, $y_2 = 0$

3. Consider the following LP: [(5)]

$$\text{Minimize:} \quad z = 2x_1 + x_2$$

$$\text{Subject to:} \quad 3x_1 + x_2 \geq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

Suppose x_1, x_2, and s_3 are a set of basic variables and the associated inverse matrix is

$$B^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{bmatrix}.$$

Using this inverse matrix, construct the entire simplex tableau (below) associated with this set of variables and check it for feasibility and optimality. (Use the space below to show your work.)

Row	Basic		RHS
0	z		
1	X_1		
2	X_2		
3	S_3		

4. Consider the following LP: [(5)]

$$\text{Maximize:} \quad z = 3x_1 + 2x_2 + 5x_3$$

$$\text{Subject to:} \quad x_1 + 2x_2 + x_3 \leq 30$$
$$3x_1 \quad\;\; + 2x_3 \leq 60$$
$$x_1 + 4x_2 \quad\;\; \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

Show that the set x_2, x_3, and s_3 is a set optimal variables using the associated inverse matrix:

$$B^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix}.$$

Use this matrix to determine the feasibility ranges of the right hand side values of each constraint.

5. A company supplies goods to three customers, who each require 30 units. The company has two ware- [(5)]
houses. Warehouse 1 has 40 units available, and warehouse 2 has 30 units available. The costs of shipping
1 unit from warehouse to customer are shown in the table below. There is a penalty for each unmet
customer unit of demand: With customer 1, a penalty case of $90 is incurred; with customer 2, $80; and
with customer 3, $110. Formulate a balanced transportation problem (i.e. create a transportation grid)
to minimize the sum of shortage and shipping costs. You do not have to solve it.

From	To		
	Customer 1	Customer 2	Customer 3
Warehouse 1	$15	$35	$25
Warehouse 2	$10	$50	$40

6. A bank has two sites at which checks are processed. Site 1 can process 10,000 checks per day and site 2   [(5)]
   can process 6,000 checks per day. The bank processes three types of checks: vendor, salary, and personal.
   The processing cost per check depends on the site, see the table below. Each day, 5,000 checks of each
   type must be processed. Formulate and solve in Excel the balanced transportation problem to minimize
   the daily cost of processing checks.

Check	Site 1	Site 2
Vendor	$0.05	$0.03
Salary	$0.04	$0.04
Personal	$0.02	$0.05

7. For each of the transportation models below, construct a starting bfs using the Norhwest-Corner,   [(5)]
   Minimum-Cost, and Vogel's method. For each model and method, compute the initial cost.

a.)

			Supply:
0	2	1	6
2	1	5	7
2	4	3	7
Demand: 5	5	10	

			Supply:
0	2	1	6
2	1	5	7
2	4	3	7
Demand: 5	5	10	

			Supply:
0	2	1	6
2	1	5	7
2	4	3	7
Demand: 5	5	10	

b.)

			Supply:
1	2	6	7
0	4	2	12
3	1	5	11
Demand: 10	10	10	

			Supply:
1	2	6	7
0	4	2	12
3	1	5	11
Demand: 10	10	10	

			Supply:
1	2	6	7
0	4	2	12
3	1	5	11
Demand: 10	10	10	

c.)

			Supply:
5	1	8	12
2	4	0	14
3	6	7	4
Demand: 9	10	11	

			Supply:
5	1	8	12
2	4	0	14
3	6	7	4
Demand: 9	10	11	

			Supply:
5	1	8	12
2	4	0	14
3	6	7	4
Demand: 9	10	11	

8. Solve Problem 5 using the Transportation Simplex Method.   [(5)]