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Instructions: All solutions should be prepared carefully and neatly. All solution sets shall be completed on this packet and submitted by uploading a scan or picture of your written work to D2L by 11:59 PM on the due date below. **Submit only a single pdf file of your entire packet. Submit any Excel or Python files as well.** The mobile app called *Genius Scan* works well. Use a PENCIL and if you make a mistake, use an eraser. Careless presentation (e.g. bad handwriting, pen scribbles, doodles, wasted space, etc) will result in a deduction of points at my discretion. Submitted work that does not demonstrate clearly the process by which one arrived at the answer will not receive credit of any kind. Academic dishonesty will not be tolerated.

Problem Set I

MAT 362-010 – Operations Research II

Due: Friday, February 16 by 11:59 PM on D2L

Read: Sections 4.14, 6.5-6.7

Solutions!

Problem Number	Available Points	Your Points
1	5	5
2	3	3
3	3	3
4	3	3
5	5	5
6	4	4
7	4	4
Total	27	27

1. A company is planning the manufacture of a product for March, April, May, and June of next year. The demand quantities are 520, 720, 520, and 620 units, respectively. The company has a steady workforce of 10 employees but can meet fluctuating production needs by hiring and firing temporary workers. The extra costs of hiring and firing a temp in any month are \$200 and \$400, respectively. A permanent worker produces 12 units per month, and a temporary worker, lacking equal experience, produces 10 units per month. The company can produce more than needed in any month and carry over the surplus to a succeeding month at a holding cost of \$50 per unit per month. Develop an optimal hiring/firing policy over the 4-month planning horizon. Formulate the LP in the space below and find a solution using the Excel Solver. Submit the Excel program and report the solution below. [(5)]

Decision Variables: for all variables, $t = 1, 2, 3, 4$.

x_t = Net number of temps at the start of month t after hiring and firing takes place. $x_t \geq 0$

S_t = Number of temps hired or fired at start of month t (S_t urs)
$= h_t - f_t$ (h_t is the # hired, f_t is the # fired) $h_t, f_t \geq 0$

i_t = units of ending inventory for month t, $i_t \geq 0$

Obj. Fun: Minimize total cost

$$Z = 50(i_1 + i_2 + i_3) + 200(h_1 + h_2 + h_3 + h_4) + 400(f_1 + f_2 + f_3 + f_4)$$

Constraints: there is always 10 permanent employees, thus the demand for each month is reduced by 120.

(Inventory)

$$i_1 = 10x_1 - 400$$
$$i_2 = 10x_2 - 600 + i_1$$
$$i_3 = 10x_3 - 400 + i_2$$
$$i_4 = 10x_4 - 500 + i_3$$
$$i_4 = 0$$

(Hire/Fire)

$$x_1 = h_1 - f_1$$
$$x_2 = x_1 + h_2 - f_2$$
$$x_3 = x_2 + h_3 - f_3$$
$$x_4 = x_3 + h_4 - f_4$$

All variables ≥ 0

Solution from Excel: $x_1 = 50, x_2 = 50, x_3 = 45, x_4 = 45$,
$i_1 = 100, i_2 = 0, i_3 = 50$,
$h_1 = 50, f_3 = 5$, the rest are 0.

$Z_{opt} = \$19{,}500$

2. Find the dual of the following LP: [(3)]

$$\text{Minimize:} \quad z = 15x_1 + 12x_2$$

$$\begin{aligned} \text{Subject to:} \quad x_1 + 2x_2 &\geq 3 \\ 2x_1 - 4x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Rewrite in equation form:

Minimize $Z = 15X_1 + 12X_2$

Subject to

$$\begin{aligned} X_1 + 2X_2 - X_3 &= 3 \\ 2X_1 - 4X_2 + X_4 &= 5 \\ X_1, X_2 &\geq 0 \end{aligned}$$

$\Rightarrow$

Dual Constraints:

$$\begin{aligned} y_1 + 2y_2 &\leq 15 \\ 2y_1 - 4y_2 &\leq 12 \\ -y_1 &\leq 0 \\ y_2 &\leq 0 \end{aligned}$$

y_1, y_2 urs

$$\boxed{\begin{array}{l} \textbf{Dual LP:} \\ \text{Maximize} \quad w = 3y_1 + 5y_2 \\ \text{Subject to} \\ \quad y_1 + 2y_2 \leq 15 \\ \quad 2y_1 - 4y_2 \leq 12 \\ \quad y_1 \geq 0,\ y_2 \leq 0 \end{array}}$$

3. Find the dual of the following LP: [(3)]

$$\begin{aligned} \text{Minimize:} \quad & z = 6x_1 + 3x_2 \\ \text{Subject to:} \quad & 6x_1 - 3x_2 + x_3 \geq 2 \\ & 3x_1 + 4x_2 + x_3 \geq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

This is a normal minimization LP. Thus, the Dual LP is a normal maximation LP.

$$\boxed{\begin{aligned} &\underline{\text{Dual LP}}: \\ &\text{Maximize} \quad w = 2y_1 + 5y_2 \\ &\text{Subject to} \quad 6y_1 + 3y_2 \leq 6 \\ &\qquad -3y_1 + 4y_2 \leq 3 \\ &\qquad y_1 + y_2 \leq 0 \\ &\qquad y_1, y_2 \geq 0 \end{aligned}}$$

4. Find the dual of the following LP: [(3)]

$$\text{Maximize:} \qquad z = 5x_1 + 6x_2$$

$$\begin{aligned} \text{Subject to:} \quad x_1 + 2x_2 &= 5 \\ -x_1 + 5x_2 &\geq 3 \\ 4x_1 + 7x_2 &\leq 8 \\ x_1 \text{ unrestricted}, x_2 &\geq 0 \end{aligned}$$

Rewrite in equation form:

Maximize $z = 5x_1^+ - 5x_1^- + 6x_2$

Subject to

$$\begin{aligned} x_1^+ - x_1^- + 2x_2 \qquad &= 5 \\ -x_1^+ + x_1^- + 5x_2 - x_3 \quad &= 3 \\ 4x_1^+ - 4x_1^- + 7x_2 \qquad + x_4 &= 8 \\ x_1^+, x_1^-, x_2 &\geq 0 \end{aligned}$$

$\Rightarrow$

Dual Constraints:

$$\begin{aligned} y_1 - y_2 + 4y_3 &\geq 5 \\ -y_1 + y_2 - 4y_3 &\geq -5 \\ 2y_1 + 5y_2 + 7y_3 &\geq 6 \\ -y_2 &\geq 0 \\ y_3 &\geq 0 \\ y_1, y_2, y_3 &\text{ urs} \end{aligned}$$

Dual LP:

Minimize $w = 5y_1 + 3y_2 + 8y_3$

Subject to

$$\begin{aligned} y_1 - y_2 + 4y_3 &= 5 \\ 2y_1 + 5y_2 + 7y_3 &\geq 6 \\ y_1 \text{ urs}, \ y_2 \leq 0, \ y_3 &\geq 0 \end{aligned}$$

5. An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, then 40 per day could be painted. If the paint shop were only painting cars, then 60 per day could be painted. If the body shop were only producing cars, then it could process 50 per day. If the body shop were only producing trucks, it could process 50 trucks per day. Each truck contributes \$300 to profit, and each car contributes \$200 to profit. The auto company need not only manufacture a single type of automobile in a single day. Formulate an LP that determines the appropriate mixture of automobile type that will maximize profits for a single day. Then, formulate the dual problem and give an interpretation of the decision variables, constraints, and objective function of the dual problem. [(5)]

Primal LP:

x_1 = # of cars manufactured

x_2 = # of trucks manufactured

Maximize $Z = 200x_1 + 300x_2$

Subject to

(painting) $\frac{1}{60}x_1 + \frac{1}{40}x_2 \le 1$

(assembling) $\frac{1}{50}x_1 + \frac{1}{50}x_2 \le 1$

* Constraint 1 is the painting constraint.
Constraint 2 is the body shop constraint.

Dual LP:

y_1 = \$ worth of painting/assembling a car /day

y_2 = \$ worth of painting/assembling a truck/day

Minimize $W = y_1 + y_2$

Subject to $\frac{1}{60}y_1 + \frac{1}{50}y_2 \ge 200$

$\frac{1}{40}y_1 + \frac{1}{50}y_2 \ge 300$

Constraint 1: total worth of processing a single car in a day

Constraint 2: total worth of processing a single truck in a day.

Obj. Fun: minimize the total cost associated to painting and assembling a single vehicle per day

6. Consider the Giapetto's Workshop problem in the text (Section 3.1, Example 1). The LP is below along with the optimal tableau. [(4)]

Maximize: $z = 3x_1 + 2x_2$

Subject to:
$$2x_1 + x_2 \le 100$$
$$x_1 + x_2 \le 80$$
$$x_1 \le 40$$
$$x_1, x_2 \ge 0$$

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	RHS
0	z	1	0	0	1	1	0	180
1	x_1	0	1	0	1	-1	0	20
2	x_2	0	0	1	-1	2	0	60
3	s_3	0	0	0	-1	1	1	20

Find the dual of the Giapetto problem. Use the optimal tableau of the Giapetto problem to determine the optimal solution of the dual problem. Verify that the Dual Theorem holds in this instance by showing the optimal solution to the dual is $\boldsymbol{c}_{BV}B^{-1}$ and $z_{opt} = w_{opt}$.

Dual LP:

Minimize $w = 100y_1 + 80y_2 + 40y_3$

Subject to
$$2y_1 + y_2 + y_3 \ge 3$$
$$y_1 + y_2 \ge 2$$
$$y_1, y_2, y_3 \ge 0$$

Optimal Dual Solution:

$y_1 = 1, y_2 = 1, y_3 = 0$

Verify Dual Theorem:

$$\vec{y} = \vec{C}_{BV}^{T} B^{-1} = \begin{bmatrix} 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \quad ✓$$

$z_{opt} = 180$ ✓

$w_{opt} = 100(1) + 80(1) + 40(0) = 180$

7. Consider the Bevco problem in the text (Section 4.12, Example 4). The LP is below. The optimal tableau, found via the Big M Method is given below as well. [(4)]

Minimize: $z = 2x_1 + 3x_2$

Subject to: $\frac{1}{2}x_1 + \frac{1}{4}x_2 \le 4$

$x_1 + 3x_2 \ge 20$

$x_1 + x_2 = 10$

$x_1, x_2 \ge 0$

Row	Basic	z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
0	z	1	0	0	0	$-\frac{1}{2}$	$\frac{1-2M}{2}$	$\frac{3-2M}{2}$	25
1	s_1	0	0	0	1	$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{5}{8}$	$\frac{1}{4}$
2	x_2	0	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5
3	x_1	0	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	5

Find the dual of the Bevco problem. Use the optimal tableau of the Bevco problem to determine the optimal solution of the dual problem. Verify that the Dual Theorem holds in this instance by showing the optimal solution to the dual is $\boldsymbol{c}_{BV}B^{-1}$ and $z_{opt} = w_{opt}$.

Dual LP:

Maximize $w = 4y_1 + 20y_2 + 10y_3$

Subject to
$\frac{1}{2}y_1 + y_2 + y_3 \le 2$
$\frac{1}{4}y_1 + 3y_2 + y_3 \le 3$
$y_1 \le 0,\ y_2 \ge 0,\ y_3$ urs

Dual Optimal Solution:

$y_1 = 0$

$y_2 = -\left(-\frac{1}{2}\right) = \frac{1}{2}$

$y_3 = \frac{3-2M}{2} + M = \frac{3}{2}$

Verify Dual Theorem:

$$\vec{y} = \vec{c}_{BV}^{\,T} B^{-1} = \begin{bmatrix} 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1/8 & -5/8 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 3/2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$ ✓

$z_{opt} = 25$ ✓

$w_{opt} = 4(0) + 20\left(\frac{1}{2}\right) + 10\left(\frac{3}{2}\right) = 10 + 15 = 25$